# Differentiability Problems

1. Determine if the following function is continuous at x=1 and differntiable at x=1

$$g(x) = \begin{cases} 8x-3, & x \leq 1 \\ 4x^2+5, & x > 1 \end{cases}$$

2. Determine if the following function is continuous at x=0 and differntiable at x=0

$$\text{Let } f(x) = \begin{cases} -x, & x \leq 0 \\ x, & x > 0 \end{cases}$$

3. **Determine the values of b and c** that make the following continuous at x=1 and differntiable at x=1

$$f(x) = \begin{cases} 3x^2+4x, & x \leq 1 \\ 2x^3+bx+c, & x > 1 \end{cases}$$

**Determine the values of a and b** that make the following continuous at x=2 and differntiable at x=2.

$$\text{Let } f(x) = \begin{cases} ax^2+10, & x < 2 \\ x^2-6x+b, & x \geq 2 \end{cases}$$

Workbook:
Read pg. 79-80
Pg. 80 Ex.1
Pg. 82 Ex. 2 (1-6)
Pg. 85 Ex. 3 (1,2)

Answers:

ote Title

① $\lim_{x\to 1^-} g(x) = 8(1) - 3 = 5$

$\lim_{x\to 1^+} g(x) = 4(1)^2 + 5 = 9$

NOT CONTINUOUS

∴ NOT DIFFERENTIABLE

② $\lim_{x\to 0^+} f(x) = 0$

$\lim_{x\to 0^-} f(x) = 0$

CONTINUOUS!

$$f'(x) = \begin{cases} -1, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

NOT DIFFERENTIABLE

$$\lim_{x\to 0^-} f'(x) \neq \lim_{x\to 0^+} f'(x)$$

③ $f(1) = 7$

$\lim_{x\to 1^-} = 3(1)^2 + 4(1) = 7$

$\lim_{x\to 1^+} = 2 + b + c$

FOR CONTINUITY:

↓

$7 = 2 + b + c$

THIS MUST BE TRUE!

$$f'(x) = \begin{cases} 6x + 4 & x \leq 1 \\ 6x^2 + b & x > 1 \end{cases}$$

$\lim_{x\to 1^-} 6(1) + 4 = 10$

$\lim_{x\to 1^+} 6(1)^2 + b = 6 + b$

FOR DIFFERENTIABILITY:

↓

$10 = 6 + b$

(MUST BE TRUE)

$\therefore \boxed{b = 4}$

$7 = 2 + b + c$

$7 = 2 + 4 + c$

$\boxed{c = 1}$

4

$$\lim_{x \to 2^-} (4a + 10)$$

$$\lim_{x \to 2^+} 4 - 12 + b$$

$$(-8 + b)$$

FOR CONTINUITY:

$$4a + 10 = -8 + b$$

MUST BE TRUE

$$f'(x) = \begin{cases} 2ax & x < 2 \\ 2x - 6 & x \geq 2 \end{cases}$$

$$\lim_{x \to 2^-} f'(x) = 4a$$

FOR DIFFERENTIABILITY:

$$4a = -2$$

$$\lim_{x \to 2^+} f'(x) = -2$$

(MUST BE TRUE)

$$\therefore \boxed{a = -\frac{1}{2}}$$

$$4a + 10 = -8 + b$$

$$-2 + 10 = -8 + b$$

$$\boxed{b = 16}$$