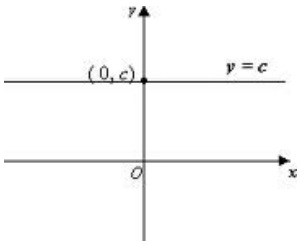
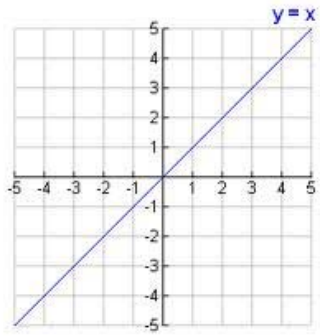
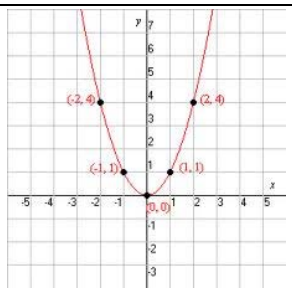
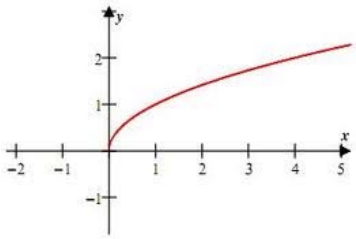
# Harold's
# Parent Functions
# "Cheat Sheet"

20 September 2016

| Function Name | Parent Function | Graph | Characteristics |
|---|---|---|---|
| **Algebra** | | | |
| **Constant** | $f(x) = c$ |  | Domain: $(-\infty, \infty)$<br>Range: [c, c]<br>Inverse Function: Undefined (asymptote)<br>Restrictions: c is a real number<br>Odd/Even: Even<br>General Form:<br>$Ay + B = 0$ |
| **Linear or Identity** | $f(x) = x$ |  | Domain: $(-\infty, \infty)$<br>Range: $(-\infty, \infty)$<br>Inverse Function:<br>$g(x) = x$<br>Restrictions: m ≠ 0<br>Odd/Even: Odd<br>General Forms:<br>$Ax + By + C = 0$<br>$y = mx + b$<br>$y - y_0 = m(x - x_0)$ |
| **Quadratic or Square** | $f(x) = x^2$ |  | Domain: $(-\infty, \infty)$<br>Range: [0, ∞)<br>Inverse Function:<br>$g(x) = \sqrt{x}$<br>Restrictions: None<br>Odd/Even: Even<br>General Form:<br>$Ax^2 + By + Cx + D = 0$ |
| **Square Root** | $f(x) = \sqrt{x}$ |  | Domain: [0, ∞)<br>Range: [0, ∞)<br>Inverse Function:<br>$g(x) = x^2$<br>Restrictions: $x \geq 0$<br>Odd/Even: Neither<br>General Form:<br>$f(x) = a\sqrt{b(x-h)} + k$ |

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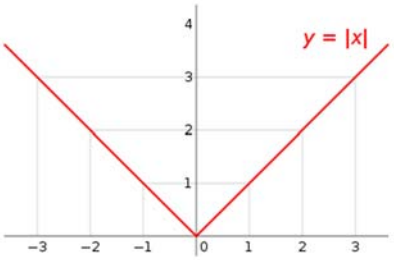
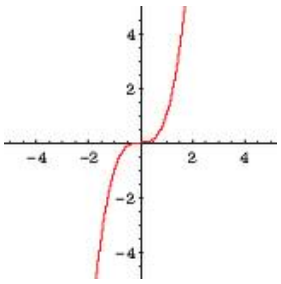
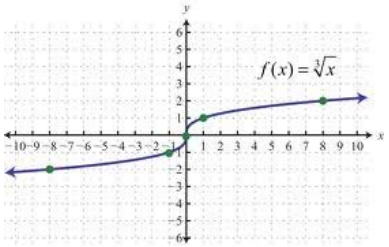
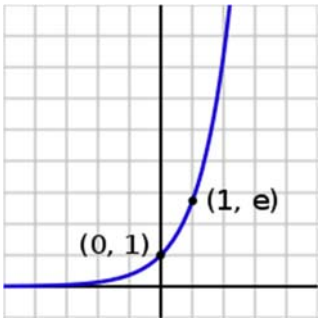
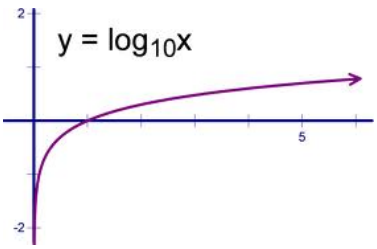
| **Absolute Value** | $f(x) = \|x\|$ |  | Domain: $(-\infty, \infty)$<br>Range: $[0, \infty)$<br>Inverse Function:<br>$f(x) = x \; for \; x \geq 0$<br>Restrictions:<br>$f(x) = \begin{cases} x, & if \; x \geq 0 \\ -x, & if \; x < 0 \end{cases}$<br>Odd/Even: Even<br>General Form:<br>$f(x) = a\|b(x-h)\| + k$ |
| **Cubic** | $f(x) = x^3$ |  | Domain: $(-\infty, \infty)$<br>Range: $(-\infty, \infty)$<br>Inverse Function:<br>$g(x) = \sqrt[3]{x}$<br>Restrictions: None<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a(b(x-h))^3 + k$ |
| **Cube Root** | $f(x) = \sqrt[3]{x}$ |  | Domain: $(-\infty, \infty)$<br>Range: $(-\infty, \infty)$<br>Inverse Function:<br>$g(x) = x^3$<br>Restrictions: None<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\sqrt[3]{b(x-h)} + k$ |
| **Exponential** | $f(x) = 10^x$<br>*or*<br>$f(x) = e^x$ |  | Domain: $(-\infty, \infty)$<br>Range: $(0, \infty)$<br>Inverse Function:<br>$g(x) = \log x$<br>*or*<br>$g(x) = \ln x$<br>Restrictions: None, x can be imaginary<br>Odd/Even: Neither<br>General Form:<br>$f(x) = a\ 10^{(b(x-h))} + k$ |
| **Logarithmic** | $f(x) = \log x$<br>*or*<br>$f(x) = \ln x$ |  | Domain: $(0, \infty)$<br>Range: $(-\infty, \infty)$<br>Inverse Function:<br>$g(x) = 10^x$<br>*or*<br>$g(x) = e^x$<br>Restrictions: x > 0<br>Odd/Even: Neither<br>General Form:<br>$f(x) = a\log(b(x-h)) + k$ |

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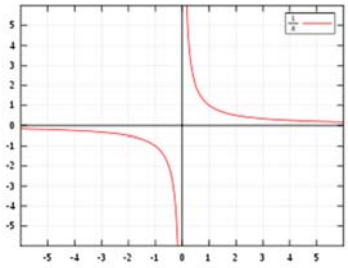
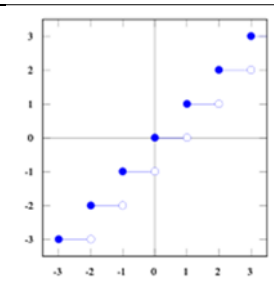
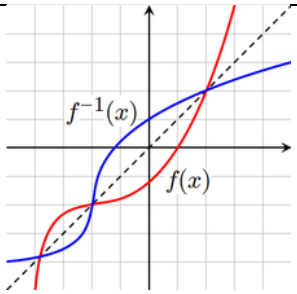
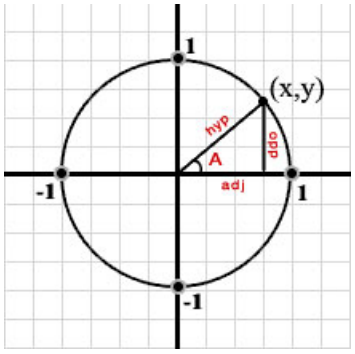
| Function Name | Parent Function | Graph | Characteristics |
|---|---|---|---|
| **Reciprocal or Rational** | $f(x) = \frac{1}{x}$ |  | Domain: $(-\infty, 0) \cup (0, \infty)$<br>Range: $(-\infty, 0) \cup (0, \infty)$<br>Inverse Function:<br>$g(x) = \frac{1}{x}$<br>Restrictions: x ≠ 0<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\left[\frac{b}{(x-h)}\right] + k$ |
| **Greatest Integer or Floor** | $f(x) = [x]$ |  | Domain: $(-\infty, \infty)$<br>Range: $(-\infty, \infty)$ whole numbers only<br>Inverse Function: Undefined (asymptotic)<br>Restrictions: Real numbers only<br>Odd/Even: Neither<br>General Form:<br>$f(x) = a[b(x-h)] + k$ |
| **Inverse Functions** | $If\ f(x) = y,$ $then$ $f^{-1}(y)$ $= f^{-1}(f(x))$ $= x$ |  | Domain of x → Domain of y<br>Range of y → Range of x<br>Inverse Function: By definition<br>Restrictions: None<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\,f(b(x-h)) + k$ |
| **Conic Sections** | | | |
| **Circle** | $x^2 + y^2 = r^2$ |  | Domain: $[-r + h,\ r + h]$<br>Range: $[-r + k,\ r + k]$<br>Inverse Function: Same as parent<br>Restrictions: None<br>Odd/Even: Both<br>Focus : $(h, k)$<br>General Forms:<br>$(x-h)^2 + (y-k)^2 = r^2$<br><br>$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$<br>$where\ A = C\ and\ B = 0$ |

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| Function Name | Parent Function | Graph | Characteristics |
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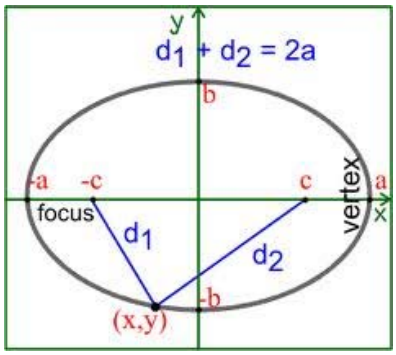
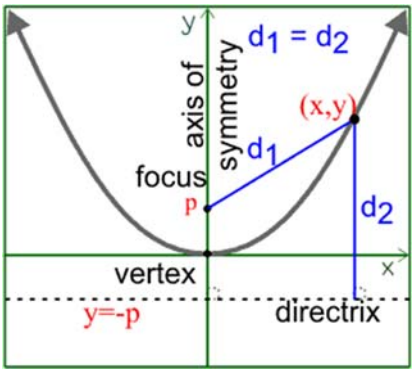
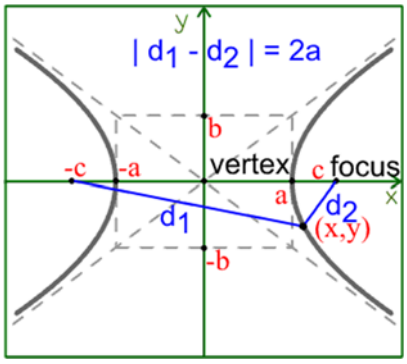
| **Ellipse** | $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ |  | Domain: $[-a+h,\ a+h]$<br>Range: $[-b+k,\ b+k]$<br>Inverse Function:<br>$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$<br>Restrictions: None<br>Odd/Even: Both<br>Foci : $c^2=a^2-b^2$<br>General Forms:<br>$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$<br>$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$<br>*where* $B^2-4AC<0$ |
| **Parabola** | $y=ax^2$ |  | Domain: $(-\infty,\infty)$<br>Range: $[k,\infty)$ or $(-\infty,k]$<br>Inverse Function:<br>$g(x)=\sqrt{x}$<br>Restrictions: None<br>Odd/Even: Even<br>Vertex : $(h,k)$<br>Focus : $(h,k+p)$<br>General Forms:<br>$(x-h)^2=4p(y-k)$<br>$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$<br>*where* $B^2-4AC=0$ |
| **Hyperbola** | $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ |  | Domain: $(-\infty,$ *-a+h*$]\cup[$*a+h*$,\infty)$<br>Range: $(-\infty,\infty)$<br>Inverse Function:<br>$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$<br>Restrictions: Domain is restricted<br>Odd/Even: Both<br>Foci : $c^2=a^2+b^2$<br>General Forms:<br>$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$<br>$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$<br>*where* $B^2-4AC>0$ |

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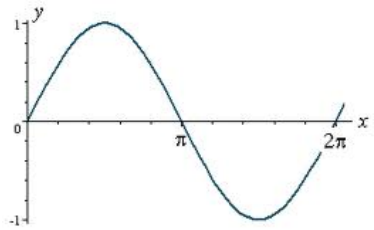
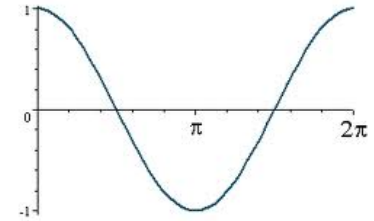
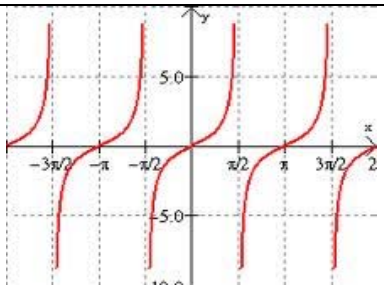
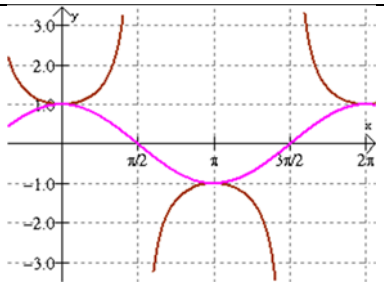
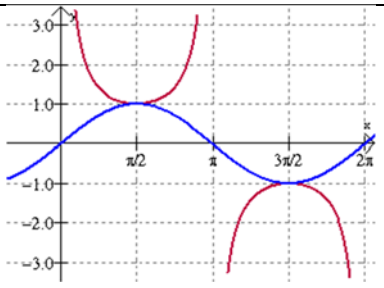
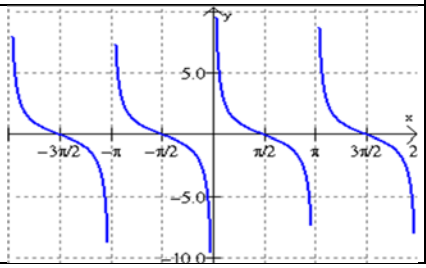
|---|---|---|---|
| **Trigonometry** | | | |
| **Sine** | $f(x) = sin\, x$ |  | Domain: $(-\infty, \infty)$<br>Range: $[-1, 1]$<br>Inverse Function: $g(x) = sin^{-1} x$<br>Restrictions: None<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ sin\ \big(b(x-h)\big) + k$ |
| **Cosine** | $f(x) = cos\, x$ |  | Domain: $(-\infty, \infty)$<br>Range: $[-1, 1]$<br>Inverse Function: $g(x) = cos^{-1} x$<br>Restrictions: None<br>Odd/Even: Even<br>General Form:<br>$f(x) = a\ cos\ \big(b(x-h)\big) + k$ |
| **Tangent** | $f(x) = tan\, x$<br>$= \frac{sin\, x}{cos\, x}$ |  | Domain: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} \pm n\pi$<br>Range: $(-\infty, \infty)$<br>Inverse Function: $g(x) = tan^{-1} x$<br>Restrictions: Asymptotes at $x = \frac{\pi}{2} \pm n\pi$<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ tan\ \big(b(x-h)\big) + k$ |
| **Secant** | $f(x) = \sec x$<br>$= \frac{1}{cos\, x}$ |  | Domain: $(-\infty, \infty)$ except for $x = \frac{\pi}{2} \pm n\pi$<br>Range: $(-\infty,-1] \cup [1, \infty)$<br>Inverse Function: $g(x) = sec^{-1} x$<br>Restrictions: Range is bounded<br>Odd/Even: Even<br>General Form:<br>$f(x) = a\ sec\ \big(b(x-h)\big) + k$ |
| **Cosecant** | $f(x) = csc\, x$<br>$= \frac{1}{sin\, x}$ |  | Domain: $(-\infty, \infty)$ except for $x = \pm n\pi$<br>Range: $(-\infty, -1] \cup [1, \infty)$<br>Inverse Function: $g(x) = csc^{-1} x$<br>Restrictions: Range is bounded<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ csc\ \big(b(x-h)\big) + k$ |
| **Cotangent** | $f(x) = cot\, x$<br>$= \frac{1}{tan\, x}$ |  | Domain: $(-\infty, \infty)$ except for $x = \pm n\pi$<br>Range: $(-\infty, \infty)$<br>Inverse Function: $g(x) = cot^{-1} x$<br>Restrictions: Asymptotes at x = $\pm n\pi$<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ cot\ \big(b(x-h)\big) + k$ |

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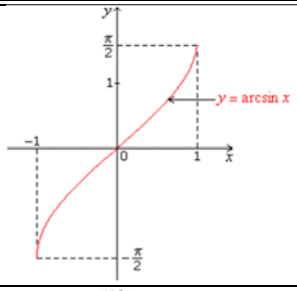
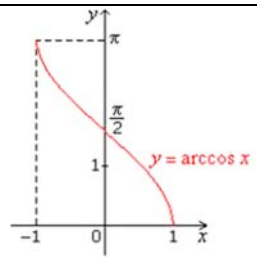
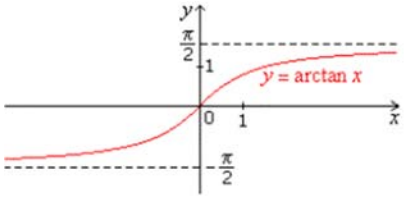
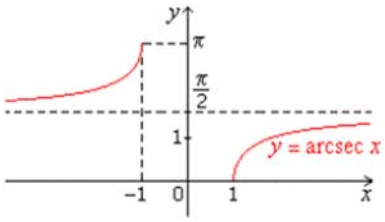
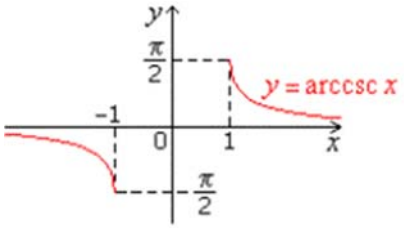
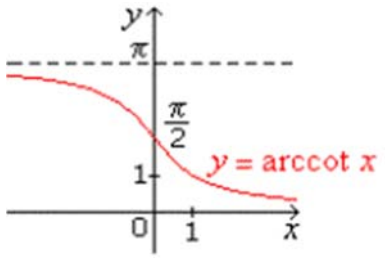
| **Arcsine** | $f(x) = sin^{-1} x$ |  | Domain: $[-1, 1]$<br>Range: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ or Quadrants I & IV<br>Inverse Function: $g(x) = sin\, x$<br>Restrictions: Range & Domain are bounded<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ sin^{-1}\left(b(x-h)\right) + k$ |
| **Arccosine** | $f(x) = cos^{-1} x$ |  | Domain: $[-1, 1]$<br>Range: $[0,\ \pi]$ or Quadrants I & II<br>Inverse Function: $g(x) = cos\, x$<br>Restrictions: Range & Domain are bounded<br>Odd/Even: None<br>General Form:<br>$f(x) = a\ cos^{-1}\left(b(x-h)\right) + k$ |
| **Arctangent** | $f(x) = tan^{-1} x$ |  | Domain: $(-\infty, \infty)$<br>Range: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ or Quadrants I & IV<br>Inverse Function: $g(x) = tan\, x$<br>Restrictions: Range is bounded<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ tan^{-1}\left(b(x-h)\right) + k$ |
| **Arcsecant** | $f(x) = sec^{-1} x$ |  | Domain: $(-\infty,-1] \cup [1, \infty)$<br>Range: $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ or Quadrants I & II<br>Inverse Function: $g(x) = sec\, x$<br>Restrictions: Range & Domain are bounded<br>Odd/Even: Neither<br>General Form:<br>$f(x) = a\ sec^{-1}\left(b(x-h)\right) + k$ |
| **Arccosecant** | $f(x) = csc^{-1} x$ |  | Domain: $(-\infty,-1] \cup [1, \infty)$<br>Range: $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$ or Quadrants I & IV<br>Inverse Function: $g(x) = csc\, x$<br>Restrictions: Range & Domain are bounded<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ csc^{-1}\left(b(x-h)\right) + k$ |
| **Arccotangent** | $f(x) = cot^{-1} x$ |  | Domain: $(-\infty, \infty)$<br>Range: $(0, \pi)$ or Quadrants I & II<br>Inverse Function: $g(x) = cot\, x$<br>Restrictions: Range is bounded<br>Odd/Even: Neither<br>General Form:<br>$f(x) = a\ cot^{-1}\left(b(x-h)\right) + k$ |

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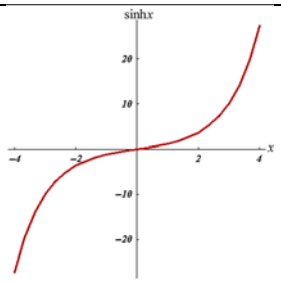
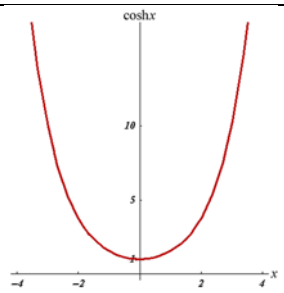
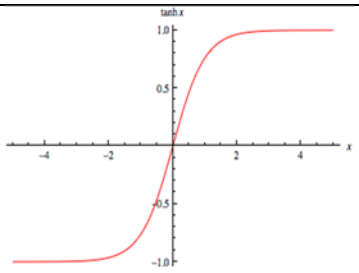
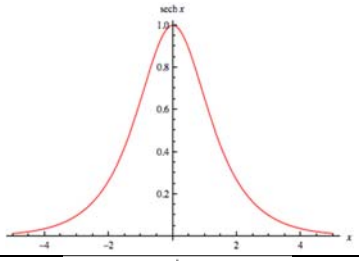
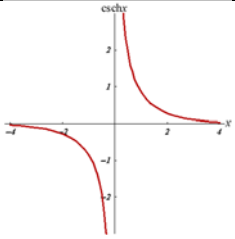
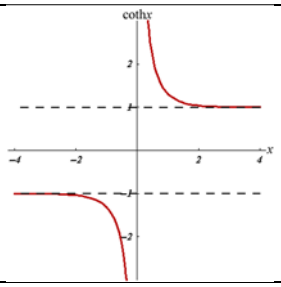
|---|---|---|---|
| **Hyperbolics** | | | |
| **Hyperbolic Sine** | $f(x) = \sinh x$ $= \frac{e^x - e^{-x}}{2}$ |  | Domain: $(-\infty, \infty)$<br>Range: $(-\infty, \infty)$<br>Inverse Function: $g(x) = sinh^{-1} x$<br>Restrictions: None<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ sinh\ \big(b(x-h)\big) + k$ |
| **Hyperbolic Cosine** | $f(x) = cosh\, x$ $= \frac{e^x + e^{-x}}{2}$ |  | Domain: $(-\infty, \infty)$<br>Range: $[1, \infty)$<br>Inverse Function: $g(x) = cosh^{-1} x$<br>Restrictions: None<br>Odd/Even: Even<br>General Form:<br>$f(x) = a\ cosh\ \big(b(x-h)\big) + k$ |
| **Hyperbolic Tangent** | $f(x) = tanh\, x$ $= \frac{e^{2x} - 1}{e^{2x} + 1}$ |  | Domain: $(-\infty, \infty)$<br>Range: $(-1, 1)$<br>Inverse Function: $g(x) = tanh^{-1} x$<br>Restrictions: Asymptotes at $y = \pm 1$<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ tanh\ \big(b(x-h)\big) + k$ |
| **Hyperbolic Secant** | $f(x) = \operatorname{sech} x$ $= \frac{1}{cosh\, x}$ |  | Domain: $(-\infty, \infty)$<br>Range: $(0, 1]$<br>Inverse Function: $g(x) = sech^{-1} x$<br>Restrictions: Asymptote at $y = 0$<br>Odd/Even: Even<br>General Form:<br>$f(x) = a\ sech\ \big(b(x-h)\big) + k$ |
| **Hyperbolic Cosecant** | $f(x) = csch\, x$ $= \frac{1}{sinh\, x}$ |  | Domain: $(-\infty, 0) \cup (0, \infty)$<br>Range: $(-\infty, 0] \cup [0, \infty)$<br>Inverse Function: $g(x) = csch^{-1} x$<br>Restrictions: Asymptotes at $x = 0, y = 0$<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ csch\ \big(b(x-h)\big) + k$ |
| **Hyperbolic Cotangent** | $f(x) = coth\, x$ $= \frac{e^{2x} + 1}{e^{2x} - 1}$ |  | Domain: $(-\infty, 0) \cup (0, \infty)$<br>Range: $(-\infty, 1) \cup (1, \infty)$<br>Inverse Function: $g(x) = coth^{-1} x$<br>Restrictions: Asymptotes at $x = 0, y = \pm 1$<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ coth\ \big(b(x-h)\big) + k$ |

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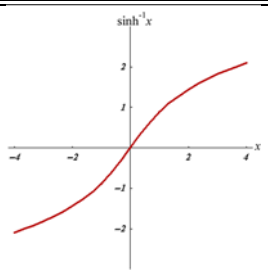
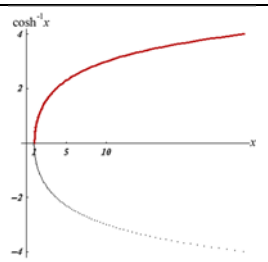
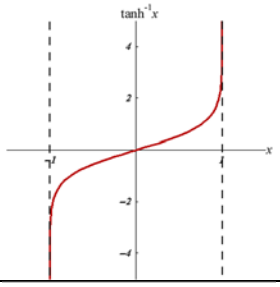
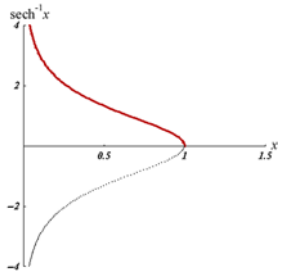
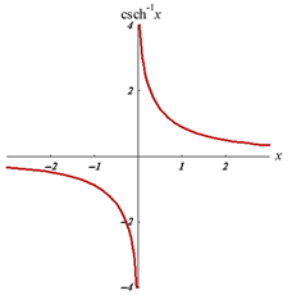
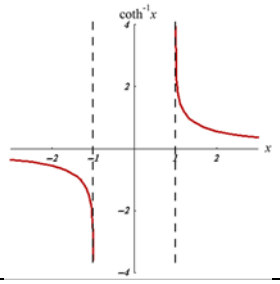
| Function Name | Parent Function | Graph | Characteristics |
|---|---|---|---|
| **Hyperbolic Arcsine** | $f(x) = sinh^{-1} x$<br>$= ln(x + \sqrt{x^2 + 1})$ |  | Domain: $(-\infty, \infty)$<br>Range: $(-\infty, \infty)$<br>Inverse Function: $g(x) = sinh\, x$<br>Restrictions: None<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ sinh^{-1}\big(b(x-h)\big) + k$ |
| **Hyperbolic Arccosine** | $f(x) = cosh^{-1} x$<br>$= ln(x + \sqrt{x^2 - 1})$ |  | Domain: $[1, \infty)$<br>Range: $[0, \infty)$<br>Inverse Function: $g(x) = cosh\, x$<br>Restrictions: $y \geq 0$<br>Odd/Even: Neither<br>General Form:<br>$f(x) = a\ cosh^{-1}\ \big(b(x-h)\big) + k$ |
| **Hyperbolic Arctangent** | $f(x) = tanh^{-1} x$<br>$= \frac{1}{2} ln\left(\frac{1+x}{1-x}\right)$ |  | Domain: $(-1, 1)$<br>Range: $(-\infty, \infty)$<br>Inverse Function: $g(x) = tanh\, x$<br>Restrictions: Asymptotes at $x = \pm 1$<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\, tanh^{-1}\ \big(b(x-h)\big) + k$ |
| **Hyperbolic Arcsecant** | $f(x) = sech^{-1} x$<br>$= ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} - 1}\right)$ |  | Domain: $(0, 1]$<br>Range: $[0, \infty)$<br>Inverse Function: $g(x) = sech\, x$<br>Restrictions:<br>Odd/Even: Neither<br>General Form:<br>$f(x) = a\ sech^{-1}\big(b(x-h)\big) + k$ |
| **Hyperbolic Arccosecant** | $f(x) = csch^{-1} x$<br>$= ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2} + 1}\right)$ |  | Domain: $(-\infty, 0) \cup (0, \infty)$<br>Range: $(-\infty, 0] \cup [0, \infty)$<br>Inverse Function: $g(x) = csch\, x$<br>Restrictions: Asymptotes at $x = 0, y = 0$<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\ csch^{-1}\big(b(x-h)\big) + k$ |
| **Hyperbolic Arccotangent** | $f(x) = coth^{-1} x$<br>$= \frac{1}{2} ln\left(\frac{x+1}{x-1}\right)$ |  | Domain: $[-\infty, -1) \cup (1, \infty]$<br>Range: $(-\infty, 0) \cup (0, \infty)$<br>Inverse Function: $g(x) = coth\, x$<br>Restrictions: Asymptotes at $x = 0, y = \pm 1$<br>Odd/Even: Odd<br>General Form:<br>$f(x) = a\, coth^{-1}\ \big(b(x-h)\big) + k$ |

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# Graphing Tips

## All Functions

| The Seven Function "Levers" | *y = a f (b (x - h)) + k* | Graphing Tips |
|---|---|---|
| 1) Move up/down ↕ | k (Vertical translation) | "+" Moves it up |
| 2) Move left/right ↔ | h (Horizontal translation) | "+" Moves it right |
| 3) Stretch up/down ↕ | a (Vertical dilation) | Larger stretches it taller or makes it grow faster |
| 4) Stretch left/right ↔ | b (Horizontal dilation) | Larger stretches it wider |
| 5) Flip about x-axis ↻ | a → –a | $f(x) \rightarrow -f(x)$<br>If $f(x) = -f(-x)$ then odd function |
| 6) Flip about y-axis ↺ | b → –b | $f(x) \rightarrow f(-x)$<br>If $f(x) = f(-x)$ then even function |
| 7) Rotate CW/CCW ↺ | $\cot 2\theta = \frac{A - C}{B}$ | "+" θ rotates CCW<br>For conic sections, where:<br>$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ |

## Trigonometric Functions

| The Six Trig "Levers" | *y = a sin (b (x - h)) + k* | Graphing Tips | Notes |
|---|---|---|---|
| 1) Move up/down ↕ | k (Vertical translation) | $k = \frac{(\max + \min)}{2}$ | If $k = f(x)$ then x-axis is replaced by $f(x)$-axis |
| 2) Move left/right ↔ | h (Phase shift) | '+' shifts right | $sin\,(x) = cos\,(x - \pi/2)$ |
| 3) Stretch up/down ↕ | a (Amplitude) | $a = \frac{(\max - \min)}{2}$ | a is NOT peak-to-peak on y-axis |
| 4) Stretch left/right ↔ | b (Frequency • 2π) | $T = \frac{2\pi}{\lvert b \rvert} = \frac{1}{f}$ | T = peak-to-peak on θ-axis<br>$T = \frac{\pi}{b}$ for $tan\,(bx)$ |
| 5) Flip about x-axis ↻ | a → –a | $f(x) \rightarrow -f(-x)$ | Odd Function: $sin\,(x) = -sin\,(-x)$ |
| 6) Flip about y-axis ↺ | b → –b | $f(x) \rightarrow f(-x)$ | Even Function: $cos\,(x) = cos\,(-x)$ |

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