Integration by Parts

Note Title 6/4/2008

When we have to integrate a function that looks like: $\int f(x)\, g(x)\ dx$

(**it's made up of two seperate functions multiplied together**) we can use the following formula

$f(x)$ $g'(x)$

ex: $(\sin x)(e^x)$

Integration by parts:

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

(PROOF THROUGH PRODUCT RULE)

- The formula may look intimidating at first glance…but it's not.
- The only thing you need to think about is: which function to assign f(x) and which is g'(x) in the original problem.
- Hint: **always assign f(x) to the function that gets simplified when you do it's derivative.**

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

Example #1

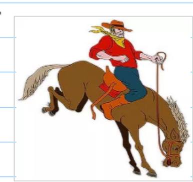
$$\int x e^x = x e^x - \int 1 e^x\, dx$$

$f(x)$ $g'(x)$

$$= x e^x - e^x + C$$

$$\boxed{\begin{aligned} f'(x) &= 1 \\ g(x) &= e^x \end{aligned}}$$

$$= e^x(x-1) + C$$



Example #2

$$\int \underset{f(x)}{x}\ \underset{g'(x)}{\sin x} = -x\cos(x) - \int (1)(-\cos x)\,dx$$

$$= -x\cos(x) - (-\sin x)$$

$f'(x) = 1$

$g(x) = -\cos x$

$$\boxed{= -x\cos x + \sin x + C}$$

EX. 3

WATCH SELECTION

$$\int \underset{g'(x)}{x}\ \underset{f(x)}{\ln x} = \frac{x^2 \ln x}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}\,dx$$

$f'(x) = \frac{1}{x}$

$$= \frac{x^2 \ln x}{2} - \int \frac{1}{2}x\ dx$$

$g(x) = \frac{1}{2}x^2$

$$\boxed{= \frac{x^2 \ln x}{2} - \frac{1}{4}x^2 + C}$$

Ex. 4

$$\int \underset{\uparrow\ f(x)}{(4x+1)}\ \underset{\uparrow\ g'(x)}{\sec^2 x} = (4x+1)(\tan x) - \int 4\tan x$$

$$= (4x+1)(\tan x) - 4\ln|\sec x|$$

$f'(x) = 4$

$g(x) = \tan x$

Ex. 5

$$\int \frac{\ln x}{x^2}$$

$$\int \underset{\uparrow\ f(x)}{\ln x} \cdot \underset{\uparrow\ g'(x)}{\frac{1}{x^2}} = -\frac{\ln x}{x} - \int \frac{1}{x}\cdot\left(-\frac{1}{x}\right)$$

$$= -\frac{\ln x}{x} - \int -\frac{1}{x^2}$$

$f'(x) = \frac{1}{x}$

$g(x) = -x^{-1}$

$$= -\frac{\ln x}{x} - x^{-1}$$

$g(x) = -\frac{1}{x}$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C$$