Intro to Integration Assignment 2020

Please record all questions and answers neatly on separate sheet

1.

Evaluate the following integrals.

(i) $\displaystyle\int \left(\frac{2}{3}e^x + \sec x \tan x\right)\, dx$

(ii) $\displaystyle\int \left((x^4 - e)^2 + 1\right)\, dx$

(iii) $\displaystyle\int (e^{3-4t} - 2\cos t)\, dt$

(iv) $\displaystyle\int \frac{u^2 + 14\sqrt{u}}{3u}\, du$

(v) $\displaystyle\int \frac{2}{2+5x}\, dx$

(vi) $\displaystyle\int \left(\sin(2x) + \sqrt[3]{x^2}\right)\, dx$

2.

Evaluate the following indefinite integrals.

(i) $\displaystyle\int x(2-x)^2\, dx$

(ii) $\displaystyle\int \frac{x^2 - x - 6}{x+2}\, dx$

(iii) $\displaystyle\int \left(4e^{2x-1} + \csc^2\left(\frac{x}{2} + \pi\right)\right)\, dx$

(iv) $\displaystyle\int \sqrt[5]{2x}\, dx$

(v) $\displaystyle\int \frac{(1 - \frac{1}{x})(2+x)}{2x}\, dx$

(vi) $\displaystyle\int (\cos(\pi x) - \sin(x-5))\, dx$

3. **BONUS: only if you have time.**

A function $f(x)$ has second derivative $f''(x) = x^2 + 1$. If $f(0) = 2$ and $f(1) = 0$, find $f(x)$.

ANSWERS:

1.

Evaluate the following integrals:

(i) $\displaystyle\int \left(\frac{2}{3}e^x + \sec x \tan x\right) dx = \frac{2}{3}e^x + \sec x + C$

(ii) $\displaystyle\int \left((x^4 - e)^2 + 1\right) dx = \int (x^8 - 2ex^4 + e^2 + 1)\, dx = \frac{1}{9}x^9 - \frac{2e}{5}x^5 + e^2x + x + C$

(iii) $\displaystyle\int (e^{3-4t} - 2\cos t)\, dt = -\frac{1}{4}e^{3-4t} - 2\sin t + C$

(iv) $\displaystyle\int \frac{u^2 + 14\sqrt{u}}{3u}\, du = \int \left(\frac{1}{3}u + \frac{14}{3}u^{-1/2}\right) = \frac{1}{6}u^2 + \frac{28}{3}u^{1/2} + C$

(v) $\displaystyle\int \frac{2}{2+5x}\, dx = \frac{2}{5}\ln|2+5x| + C$

(vi) $\displaystyle\int \left(\sin(2x) + \sqrt[3]{x^2}\right) = \int \left(\sin(2x) + x^{2/3}\right) = -\frac{1}{2}\cos(2x) + \frac{3}{5}x^{5/3} + C$

2.

Evaluate the following indefinite integrals.

(i) $\int x(2-x)^2\,dx = \int x(4-4x+x^2)\,dx = \int (4x-4x^2+x^3)\,dx = 2x^2-\frac{4}{3}x^3+\frac{1}{4}x^4+$

(ii) $\int \frac{x^2-x-6}{x+2}\,dx = \int \frac{x-3)(x+2)}{x+2}\,dx = \int (x-3)\,dx = \frac{1}{2}x^2 - 3x + C$

(iii)

$$\begin{aligned}\int \left(4e^{2x-1} + \csc^2\left(\frac{x}{2}+\pi\right)\right)\,dx &= 4\left(\frac{1}{2}e^{2x-1} - 2\cot\left(\frac{x}{2}+\pi\right)\right) + C \\ &= 2e^{2x-1} - 2\cot\left(\frac{x}{2}+\pi\right) + C\end{aligned}$$

(iv) $\int \sqrt[5]{2x}\,dx = \int (2x)^{1/5} = \int 2^{1/5}x^{1/5} = 2^{1/5}\left(\frac{5}{6}\right)x^{6/5} + C = \frac{5\cdot 2^{1/5}}{6}x^{6/5} + C$

(v)

$$\begin{aligned}\int \frac{(1-\frac{1}{x})(2+x)}{2x}\,dx &= \int \frac{(2+x-\frac{2}{x}-1}{2x}\,dx \\ &= \int \frac{1+x-\frac{2}{x}}{2x}\,dx \\ &= \int \left(\frac{1}{2x}+\frac{1}{2}-\frac{2}{x^2}\right)\,dx \\ &= \frac{1}{2}\ln|x| + \frac{1}{2}x + \frac{2}{x} + C\end{aligned}$$

(vi) $\int (\cos(\pi x) - \sin(x-5))\,dx = \frac{1}{\pi}\sin x + \cos(x-5) + C$

3.

A function $f(x)$ has second derivative $f''(x) = x^2 + 1$. If $f(0) = 2$ and $f(1) = 0$, find $f(x)$.

We have that $f'(x) = \int f''(x)\,dx = \int (x^2+1)\,dx = \frac{1}{3}x^3 + x + C$, and so $f(x) = \int f'(x)\,dx = \int (\frac{1}{3}x^3 + x + C)\,dx = \frac{1}{12}x^4 + \frac{1}{2}x + Cx + D$, for some constants C and D. Now, $f(0) = 2$ implies that $D = 2$, and so $f(x) = \frac{1}{12}x^4 + \frac{1}{2}x + Cx + 2$. Since $f(1) = 0$, we obtain the equation $\frac{1}{12} + \frac{1}{2} + C + 2 = 0$, or $C + \frac{31}{12} = 0$. Hence $C = -\frac{31}{12}$, and so

$$f(x) = \frac{1}{12}x^4 + \frac{1}{2}x - \frac{31}{12}x + 2.$$