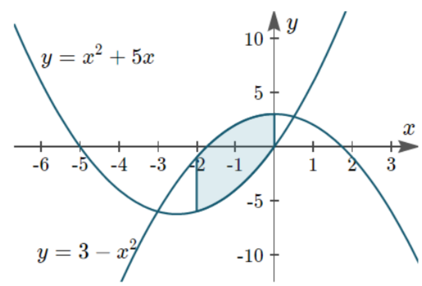
Note Title

5/6/20

# AREA BETWEEN TWO CURVES

**Ex. 1** Find the area between the curves $y = x^2 + 5x$ and $y = 3 - x^2$ between $x = -2$ and $x = 0$.

REMEMBER UPPER - LOWER




$$\int_{-2}^{0} 3 - x^2 - (x^2 + 5x)\, dx$$

$$= \int_{-2}^{0} -2x^2 - 5x + 3\, dx$$

$$= \int_{-2}^{0} \left[(-2x^2 - 5x + 3)\right] dx$$

$$= \left[-\frac{2}{3}x^3 - \frac{5}{2}x^2 + 3x\right]_{-2}^{0}$$

$$= 0 - \left[\frac{16}{3} - 10 - 6\right]$$

$$= 10\frac{2}{3} \text{ sq units}$$

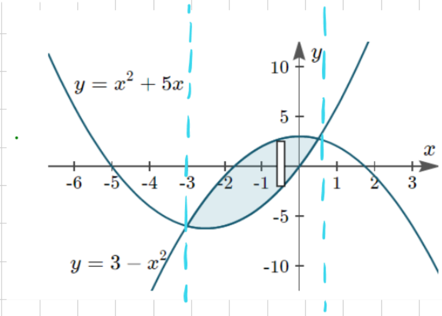
Ex.2

Find the area bounded by the curves

$y = x^2 + 5x$ and $y = 3 - x^2$.

HERE THE BOUNDARIES ARE WHERE THE FUNCTIONS INTERSECT.

SO WE NEED TO FIND a, b




$$\int_a^b f(x)\,dx$$

THIS IS DONE BY EQUATING THE TWO FUNCTIONS

$$x^2 + 5x = 3 - x^2$$

Points of intersection occur where:

$x^2 + 5x = 3 - x^2$

$2x^2 + 5x - 3 = 0$

$(x + 3)(2x - 1) = 0$

So $x = -3$ or $x = 0.5$

NOW:

$$\text{AREA} = \int_{-3}^{0.5} \text{upper} - \text{lower}\ dx$$

$$= \int_{-3}^{0.5} \left([3 - x^2] - [x^2 + 5x]\right) dx$$

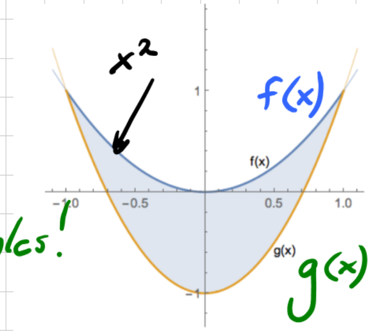
$$= \int_{-3}^{0.5} (3 - 5x - 2x^2)\, dx$$

$$= \left[3x - \frac{5x^2}{2} - \frac{2x^3}{3}\right]_{-3}^{0.5}$$

$= 14.29$ sq units

EX.3 find the area between:

$$y = 2x^2 - 1 \text{ and } y = x^2$$




THIS IS NO DIFFERENT THEN PREVIOUS EXAMPLES!

JUST INTEGRATE $\displaystyle\int \text{TOP} - \text{BOTTOM}$

$\qquad\qquad f(x) - g(x)$

FIND INTERCEPTS:

$$2x^2 - 1 = x^2$$

$$x^2 = 1$$

$$x = -1, 1$$

$$\text{AREA} = \int_{-1}^{1} x^2 - (2x^2 - 1)\, dx$$

$$= \int_{-1}^{1} -x^2 + 1\, dx = -\frac{1}{3}x^3 + x \Big]_{-1}^{1}$$

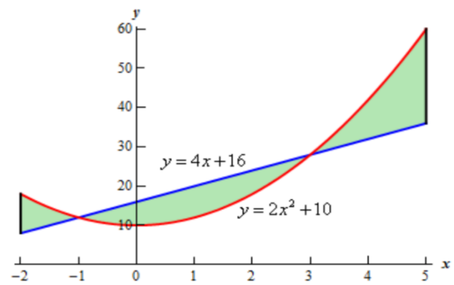
$$= \left(-\frac{1}{3} + 1\right) - \left(\frac{1}{3} - 1\right)$$

$$= \frac{2}{3} - \left(-\frac{2}{3}\right) = \frac{4}{3} \text{ UNITS}^2$$

EX. 4

Determine the area of the region bounded by $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$ and $x = 5$.

- THE OLD SWITCHEROO!
- SOMETIMES THE "UPPER" AND "LOWER" FUNCTIONS SWAP.




INTERCEPTS:

$$4x + 16 = 2x^2 + 10$$

$$0 = 2x^2 - 4x - 6$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$x = -1, 3$

3 SEPERATE INTEGRALS!

$$A = \int_{-2}^{-1} 2x^2 + 10 - (4x + 16)\ dx + \int_{-1}^{3} 4x + 16 - \left(2x^2 + 10\right)\ dx + \int_{3}^{5} 2x^2 + 10 - (4x + 16)\ dx$$

$$= \int_{-2}^{-1} 2x^2 - 4x - 6\,dx + \int_{-1}^{3} -2x^2 + 4x + 6\,dx + \int_{3}^{5} 2x^2 - 4x - 6\,dx$$

$$= \left(\frac{2}{3}x^3 - 2x^2 - 6x\right)\Bigg|_{-2}^{-1} + \left(-\frac{2}{3}x^3 + 2x^2 + 6x\right)\Bigg|_{-1}^{3} + \left(\frac{2}{3}x^3 - 2x^2 - 6x\right)\Bigg|_{3}^{5}$$

$$= \frac{14}{3} + \frac{64}{3} + \frac{64}{3}$$

$$= \frac{142}{3}$$