SECOND DERIVATIVE TEST EXAMPLES

→ USE THE FIRST AND SECOND DERIVATIVE TESTS TO FIND OUT THE CRITICAL POINTS OF THE FOLLOWING FUNCTION A IF THEY ARE MAX or MIN's

$$f(x) = \frac{1}{3}x^3 - x$$

$$f'(x) = \frac{3}{3}x^2 - 1$$

$$f'(x) = x^2 - 1$$

$$0 = x^2 - 1 \quad \text{SET } f'(x) = 0$$

$$0 = (x+1)(x-1)$$

CRITICAL POINTS @ $x = -1, 1$
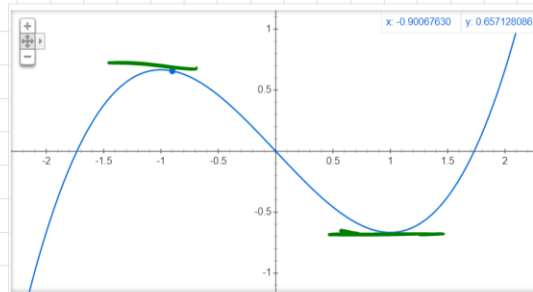
$$f'(x) = x^2 - 1$$

$$f''(x) = 2x \quad \text{SECOND DERIVATIVE}$$

$$f''(-1) = -2 \quad (\text{CONCAVE DOWN @ } x = -1)$$

$$f''(1) = 2 \quad (\text{CONCAVE UP @ } x = 2)$$

MAX @ $x = -1$
MIN @ $x = 1$

EXAMPLE #2

USE THE FIRST AND SECOND DERIVATIVE TO FIND
THE LOCAL MAX AND mins AND INFLECTION POINTS
ON THE FOLLOWING FUNCTION

$$f(x) = x^4 - 4x^3$$

$$f'(x) = 4x^3 - 12x^2$$

$$f'(x) = 4x^2(x-3)$$

$$0 = 4x^2(x-3)$$

$$f'(x) = 4x^3 - 12x^2$$

$$f'(x) = 4x^2(x-3)$$

$$0 = 4x^2(x-3)$$

CRITICAL POINTS @ $x = 0$
$x = 3$ } max or mins?

SECOND DERIVATIVE TEST

$$f'(x) = 4x^3 - 12x^2$$

$$f''(x) = 12x^2 - 24x$$

$$f''(x) = 12x(x-2)$$

$$f''(0) = 0$$ INFLECTION POINT (NOT A MAX or MIN @ $x=0$)

$$f''(3) = 12(3)(3-2)$$
$$f''(3) = 36 \quad (POSITIVE) \quad +\,+$$ MIN @ $x = 3$

INFLECTION POINTS

$$f''(x) = 12x(x-2) \quad SET \ f''(x) = 0$$

$$0 = 12x(x-2)$$

INFLECTION POINTS @ $x = 0, 2$