

Commerce Optimization Problems

① PROFIT = REVENUE - COST

$$P = R(x) - C(x) \quad \text{WHERE } x = \# \text{ OF PASSES}$$

$$P = (2x^3 + 40x^2 + 8x) - (3x^3 + 19x^2 + 80x - 800)$$

$$P = -x^3 + 21x^2 - 72x - 800$$

$$P' = -3x^2 + 42x - 72 \quad \text{DERIVATIVE}$$

$$0 = -3(x^2 - 14x + 24) \quad \text{SET TO ZERO (AND FACTOR)}$$

$$0 = x^2 - 14x + 24$$

$$0 = (x - 12)(x - 2)$$

MAX OF MIN @ $x = 12, 2$ (POSSIBLE) CHECK

$$P'' = -6x + 42$$

SECOND DERIVATIVE TEST

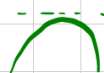
$$P''(2) = -12 + 42$$

$$P''(12) = -6(12) + 42$$

$$P''(2) = \text{POSITIVE}$$

$$P''(12) = -30 \text{ (NEGATIVE VALUE!)} \quad \text{MAX}$$

MIN 



∴ WB SHOULD AIM TO SELL

12000 SEASONS PASSES!

#2

$$\text{PROFIT} = \text{REVENUE} - \text{COST}$$

$$P = (\# \text{ OF SHIRTS}) (\text{SELLING PRICE}) - (\text{COST TO PRODUCE } x \text{ SHIRTS})$$

LET $x = \# \text{ OF SHIRTS}$

$$P = x(30 - 0.2\sqrt{x}) - (500 + 9x)$$

$$P = 30x - 0.2x^{3/2} - 500 - 9x$$

$$P = -0.2x^{3/2} + 21x - 500$$

$$P' = -0.3x^{1/2} + 21$$

$$0 = -0.3\sqrt{x} + 21$$

$$0.3\sqrt{x} = 21$$

$$\sqrt{x} = 70$$

$$x = 4900 \text{ SHIRTS}$$

b) SELLING PRICE

$$S = 30 - 0.2\sqrt{x}$$

$$S = 30 - 0.2(70)$$

$$S = 30 - 14$$

$$S = 16 \text{ } \$16.00$$

c) PROFIT = REVENUE - COST

$$P = x(30 - 0.2\sqrt{x}) - (500 + 9x)$$

$$P = 4900(30 - 0.2(70)) - (500 + 9(4900))$$

$$P = \$33800$$

"ORHARD" PROBLEMS (MORE ITEMS, LESS YIELD)

ATTRACT → (MORE CUSTOMERS, CHEAPER PRICE)



Problem 1. The regular air fare between Boston and San Francisco is \$500. An airline using planes with a capacity of 300 passengers on this route observes that they fly with an average of 180 passengers. Market research tells the airlines' managers that each \$ 5 fare reduction would attract, on average, 3 more passengers for each flight. How should they set the fare to maximize their revenue? Explain your reasoning to receive credit.

USUALLY EASIEST TO MAKE UNKNOWN VARIABLE (ADDITIONAL UNITS)

n = GROUPS OF 3 ADDITIONAL PASSENGERS

PROFIT = PRICE × (# OF PASSENGERS)

PRICE = $500 - 5(n)$ (MAKE SENSE? TEST IT OUT!)

OF PASSENGERS = $180 + 3n$ (MAKE SENSE? TEST IT OUT!)

PROFIT = $(500 - 5n)(180 + 3n)$

PROFIT = $90000 + 1500n - 900n - 15n^2$

PROFIT = $90000 + 600n - 15n^2$

NOW DO DERIVATIVE AND SET TO ZERO :

$P' = 600 - 30n$

$0 = 600 - 30n$

$-600 = -30n$

$n = 20$

REMEMBER n = GROUPS OF 3 OVER 180

SO 60 ADDITIONAL PASSENGERS OVER 180

PUT 240 PASSENGERS ON PLANE FOR MAX PROFIT

PRICE = $500 - 5n$

SET PRICE @ $500 - 5(20)$

\$ 400

Problem 3. A Florida Citrus grower estimates that if 60 orange trees are planted; the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

REMEMBER MAKE n (ADDITIONAL TREES)

$$\begin{aligned} \text{ORANGES PRODUCED BY EACH TREE} &= (400 - 4n) \\ \text{\# OF TREES} &= (60 + n) \end{aligned}$$



$$\text{TOTAL YIELD OF ORANGES} = (400 - 4n)(60 + n)$$

$$= 24000 + 400n - 240n - 4n^2$$

$$y = 24000 + 160n - 4n^2$$

$$y' = 160 - 8n \quad \text{DERIVATIVE}$$

(SET TO ZERO)

$$0 = 160 - 8n$$

$$8n = 160$$

$$n = 20$$

ADD 20 TREES:

$$40 + 20 \dots$$

you WANT 60 TREES!