Commerce Optimization Problems

① PROFIT = REVENUE − COST

$P = R(x) - C(x)$ WHERE x = # of PASSES

$P = (2x^3 + 40x^2 + 8x) - (3x^3 + 19x^2 + 80x - 800)$

$P = -x^3 + 21x^2 - 72x - 800$

$P' = -3x^2 + 42x - 72$ DERIVATIVE

$0 = -3(x^2 - 14x + 24)$ SET TO ZERO (AND FACTOR)

$0 = x^2 - 14x + 24$

$0 = (x - 12)(x - 2)$

MAX OR MIN ≡ @ $x = 12, 2$ (POSSIBLE) CHECK

SECOND DERIVATIVE TEST

$P'' = -6x + 42$

$P''(12) = -6(12) + 42$

$P''(12) = -30$ (NEGATIVE VALUE!) MAX

$P''(2) = -12 + 42$

$P''(2) =$ POSITIVE MIN

∴ WB SHOULD AIM TO SELL

12 000 SEASONS PASSES!

#2

$\$\$\$$

PROFIT = REVENUE − COST

$P = (\#\text{ of SHIRTS})(\text{SELLING PRICE}) - (\text{COST TO PRODUCE } x \text{ SHIRTS})$

LET x = # OF SHIRTS

$$P = x(30 - 0.2\sqrt{x}) - (500 + 9x)$$

$$P = 30x - 0.2x^{3/2} - 500 - 9x$$

$$P = -0.2x^{3/2} + 21x - 500$$

$$P' = -0.3x^{1/2} + 21$$

$$0 = -0.3\sqrt{x} + 21$$

$$0.3\sqrt{x} = 21$$

$$\sqrt{x} = 70$$

$$\boxed{x = 4900} \text{ SHIRTS}$$

b) SELLING PRICE

$$S = 30 - 0.2\sqrt{x}$$

$$S = 30 - 0.2(70)$$

$$S = 30 - 14$$

$$\boxed{S = 16} \quad \$16.00$$

c) PROFIT = REVENUE − COST

$$P = x(30 - 0.2\sqrt{x}) - (500 + 9x)$$

$$P = 4900(30 - 0.2(70)) - (500 + 9(4900))$$

$$\boxed{P = \$33800}$$

lote Title

"ORHARD" PROBLEMS (MORE ITEMS, LESS YEILD)

ATTRACT → (MORE CUSTOMERS, CHEAPER PRICE)

Problem 1. The regular air fare between Boston and San Francisco is $500. An airline using planes with a capacity of 300passengers on this route observes that they fly with an average of 180 passengers. Market research tells the airlines' managers that each $ 5 fare reduction would attract, on average, 3 more passengers for each flight. How should they set the fare to maximize their revenue? Explain your reasoning to receive credit.

USUALLY EASIEST TO MAKE UNKNOWN VARIABLE (ADDITIONAL UNITS)

n = GROUPS OF 3 ADDITIONAL PASSANGERS

PROFIT = PRICE x (# OF PASSANGERS)

PRICE = $500 - 5(n)$ (MAKE SENSE? TEST IT OUT!)

OF PASSANGERS = $180 + 3n$ (MAKE SENSE? TEST IT OUT!)

PROFIT = $(500 - 5n)(180 + 3n)$

PROFIT = $90\,000 + 1500n - 900n - 15n^2$

PROFIT = $90\,000 + 600n - 15n^2$

NOW DO DERIVATIVE AND SET TO ZERO:

$P' = 600 - 30n$

$0 = 600 - 30n$

$-600 = -30n$

$n = 20$

REMEMER n = GROUPS OF 3 OVER 180

SO 60 ADDITIONAL PASSANGERS OVER 180

PUT 240 PASSANGERS ON PLANE FOR MAX PROFIT

PRICE = $500 - 5n$

SET PRICE @ $500 - 5(20)$

$ 400

Problem 3. A Florida Citrus grower estimates that if 60 orange trees are planted; the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

REMEMBER MAKE n (ADDITIONAL TREES)

ORANGES PRODUCED BY EACH TREE $= (400 - 4n)$

OF TREES $= (60 + n)$

TOTAL YEILD OF ORANGES $= (400 - 4n)(60 + n)$

$= 24000 + 400n - 240n - 4n^2$

$y = 24000 + 160n - 4n^2$

$y' = 160 - 8n$ DERIVATIVE

(SET TO ZERO)

$0 = 160 - 8n$

$8n = 160$

$n = 20$

ADD 20 TREES:

40 + 20 .....

YOU WANT 60 TREES!