

Integration Self TEST

1. Evaluate:

$$\sum_{i=0}^3 (5 + \sqrt{4^i})$$

Answer: 35

2. Write the sigma notation for the following series:

$$1 + 5 + 9 + \dots + 193 + 197$$

Answer:

$$\sum_{k=1}^{50} (4k - 3)$$

3. In the equation to the right the limit as n approached infinity describes:

- a) An infinite distance between a and b
- b) Making Δx as big as possible
- c) An infinite number of integrals
- d) An infinite number of rectangles

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \left[\sum_{i=1}^n f(x_i) \Delta x \right]$$

4. Find the area under x^2 from 1 to 9 using 4 rectangles

Answer: 328

5.

Estimate the area under the graph of $f(x) = 1/x$ from $x = 1$ to $x = 4$

Use $n=6$

Answer: 1.218 units²

6.

11. $\int (3x - 1)^2 dx$

12. $\int \left(x - \frac{1}{x}\right)^2 dx$

13. $\int x(2x + 3) dx$

14. $\int (2x - 5)(3x + 1) dx$

15. $\int \frac{8x - 5}{\sqrt[3]{x}} dx$

16. $\int \frac{2x^2 - x + 3}{\sqrt{x}} dx$

Answers:

11. $3x^3 - 3x^2 + x + C$

12. $\frac{x^3}{3} - 2x - x^{-1} + C$

13. $\frac{2x^3}{3} + \frac{3x^2}{2} + C$

$$14. 2x^3 - \frac{13x^2}{2} - 5x + C$$

$$15. \frac{24x^{5/3}}{5} - \frac{15x^{2/3}}{2} + C$$

$$16. \frac{4x^{5/2}}{5} - \frac{2x^{3/2}}{3} + 6x^{1/2} + C$$

7.

$$71. \int_{-\pi/2}^{\pi/2} (2t + \cos t) dt$$

$$72. \int_1^e \left(2x + \frac{1}{x} \right) dx$$

$$73. \int_1^5 \frac{x+1}{x} dx$$

$$74. \int_0^2 (e^x + 6) dx$$

$$75. \int_0^3 (t - e^t) dt$$

$$76. \int_{-1}^1 (e^\theta + \sin \theta) d\theta$$

$$77. \int_e^{2e} \left(\cos x - \frac{1}{x} \right) dx$$

$$71. 2$$

$$72. e^2$$

$$73. 4 + \ln 5$$

$$74. e^2 + 11$$

$$75. \frac{11}{2} - e^3$$

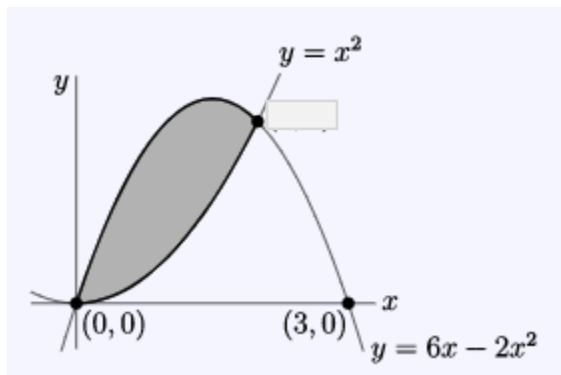
$$76. e - e^{-1}$$

$$77. \sin(2e) - \sin(e) - \ln(2)$$

8.

Find the area of the finite region bounded by $y = x^2$ and $y = 6x - 2x^2$.

First find the intercepts

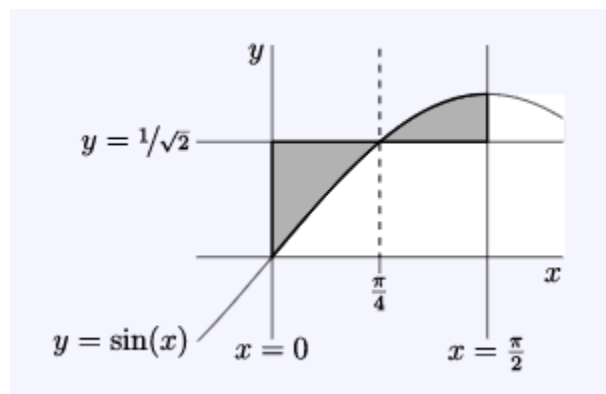


Answer: 4

9.

Find the area between the curves $y = \frac{1}{\sqrt{2}}$ and $y = \sin(x)$ with x running from 0 to $\frac{\pi}{2}$.

Notice that the top and bottom functions swap at $\pi/4$



Answer:

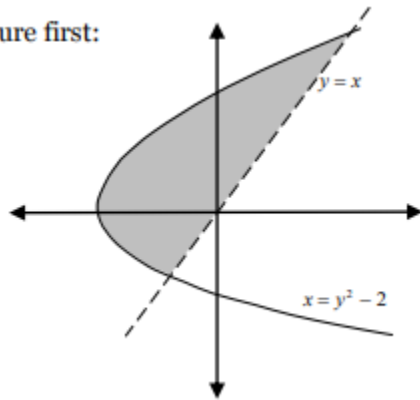
$$= \sqrt{2} - 1$$

10.

Find the area of the region enclosed by the following curves: $x_1 = y^2 - 2$, and $x_2 = y$.

- Find intercepts first
- Integrate with respect to y

picture first:



Use desmos or wolfram alpha to check answer.

11.

Tough one:

Find the area bound by

$$y = -2 \cdot \sec^2 x$$

$$y = 2\cos x$$

$$x = 0$$

$$x = \frac{\pi}{4}$$

Answer: 3.414