Another Cool Calculus Application!

Remember, learning about Calculus is **cool** because it can be used to solve all sorts of practical problems. In this course we will look at two kinds of problems that can be solved by **derivatives**:

1. Optimization Problems
2. **Related Rate Problems**

RELATED RATES:

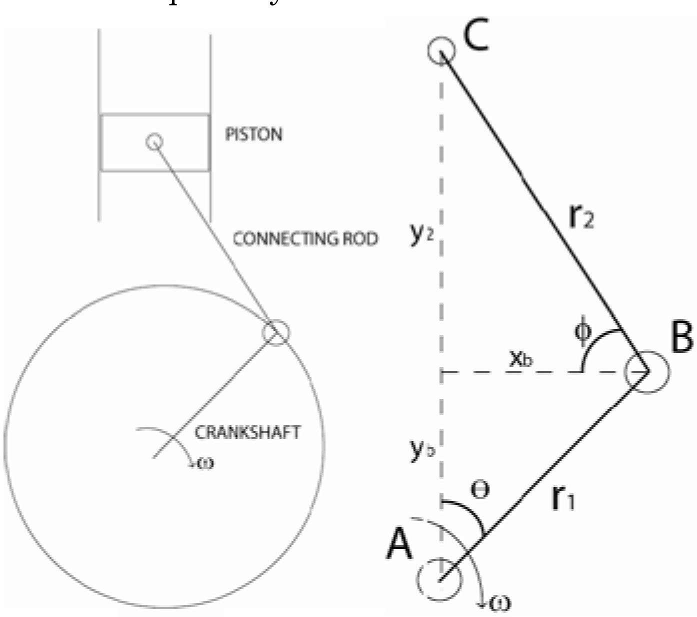
All sorts of quantities can change over time. Temperature, speed, volumes, distances…you name it.

Sometimes the *rate* at which a quantity changes is **RELATED TO** how another quantity changes:

<u>Examples</u>:

- The **rate** at which the volume of a balloon increased (as it is being blown up) is **REALATED TO** the rate at which its radius increases
- The **rate** at which the *area* of a pancake increases (as it is being poured into a pan) is **REALATED** to the rate at which its *diameter* is increasing.

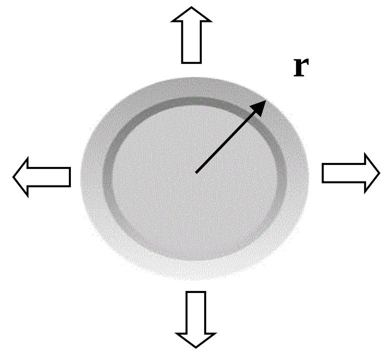
Knowing some calculus and the rate increase of a single quantity, we can determine the rate increase of <u>many other quantities</u> that are **RELATED TO** the initial quantity!



*The <u>rate</u> at which the **piston** moves down is <u>related</u> to the <u>rate</u> at which the **crankshaft** turns*

RELATED RATES!

Fun Notation you need to know:



Imagine a plate that is put in an oven. Because of the heat it, expands and it's radius increases at a constant rate of **0.045cm/minute**. → $\frac{dr}{dt}$

This is a **rate of change of radius with respect to time.** → $\frac{dr}{dt}$

If we wanted to know how the **Area of the plate changes with respect to time** we will have to do some calculus. → $\frac{dA}{dt}$

Note that: $\frac{dA}{dt}$ Will **NOT** be the same as $\frac{dr}{dt}$

And in fact it **won't even be constant** like $\frac{dr}{dt}$

We **can** find the **rate of change of Area** $\frac{dA}{dt}$ by doing the following:

1. First we **link** the two quantities of interest with a known equation:

   In this case: $A = \pi r^2$ (Area of a circle in terms of radius).

2. We **differentiate** "implicitly" with respect to **time**.

$$A = \pi r^2$$

$$\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$$

$$\frac{dA}{dt} = \pi 2 r \frac{dr}{dt}$$

$$\boxed{\frac{dA}{dt} = 2\pi r \frac{dr}{dt}}$$

REVIEW THIS

WE USE IMPLICIT DIFFERENTIATION WITH RESPECT TO TIME

CHAIN RULE....
BRING THE 2 DOWN
DO THE DIRIVATIVE
OF WHAT'S INSIDE
WITH RESPECT TO TIME.

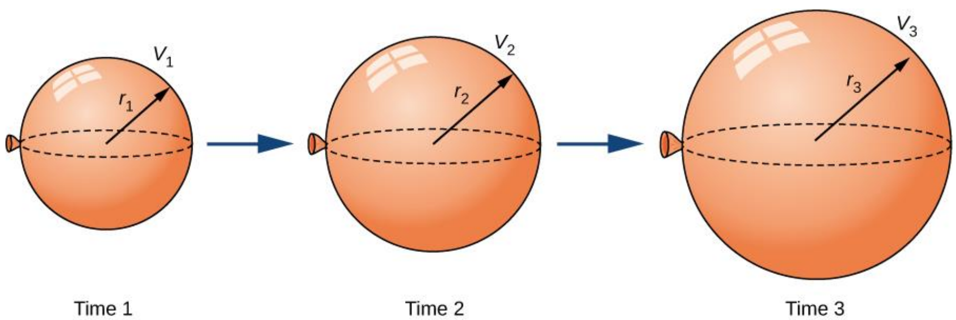
Now we have discovered the true relationship between $\frac{dA}{dt}$ and $\frac{dr}{dt}$ and can use it to solve for the rate of change of the area.

EXAMPLE #1

Imagine Alex is inflating a giant balloon for Abby's Birthday. They borrow Diesel's pump (which can provide a constant air flow of **1000cm³/s**). As the balloon grows close to having a 50cm radius, Cain becomes concerned that the radius of the balloon may be increasing too fast!

Cain decides to calculate ***the rate at which the balloon's radius is increasing*** **when the radius of the balloon reaches 50cm.**

[note the packaging on the balloon states that the balloon's radius must not increase at a rate greater than 0.029cm/s or it will break. Will the students be safe?



Example #2

Ben and Marine are painting Jesse's house. Ben climbs a 13 ft ladder before properly securing it. The ladder begins to slide down the side of the house. If the **top** of the ladder slides down at a **constant** rate of 2ft/s, determine how quickly the **base** of ladder will be sliding *horizontally away from the house* when the top of the ladder is 5 ft from the ground.