WARNING! WATCH FOR UNUSUAL CRITICAL POINTS

RECALL → CRITICAL POINTS WHEN $f'(x) = 0$

$\boxed{f'(x) = D.N.E}$

$f'(x)$ DOES NOT EXIST WHEN

$f'(x) = \dfrac{\text{Any\#}}{0}$ ← ZERO ON DENOMINATOR

Example # 1 (CUSP)

$f(x) = (x-4)^{2/3}$ ← CLUE

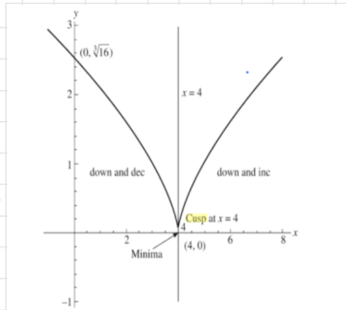


Figure 4.16 Graph of $y = (x-4)^{2/3}$.

$f'(x) = \frac{2}{3}(x-4)^{-\frac{1}{3}}$

$f'(x) = \dfrac{2}{3(x-4)^{\frac{1}{3}}}$

CAN'T MAKE $f'(x) = 0$, BUT BOTTOM IS ZERO IF $x = 4$

(CRITICAL POINT @ 4)

IN THIS CASE DON'T USE SECOND DERIVATIVE TEST

$f''(x)$ WON'T EXIST @ $x = 4$

SO TEST $f'(x)$ BEFORE AND AFTER $x = 4$

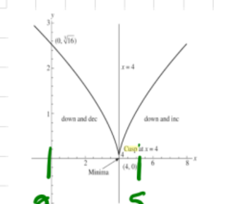


Figure 4.16 Graph of $y = (x-4)^{2/3}$.

$f'(x) = \dfrac{2}{3(x-4)^{\frac{1}{3}}}$

$f'(0) = \dfrac{2}{3(-4)^{\frac{1}{3}}} = -$ NEGATIVE #

$f'(5) = \dfrac{2}{3(1)} = +$ POSITIVE #

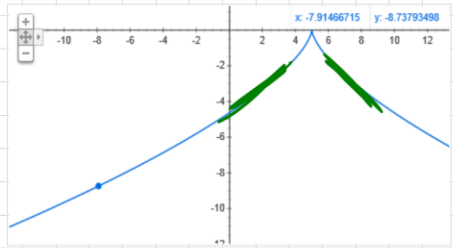
MUST BE A MIN!

Example#2

$$f(x) = -(-2x+10)^{2/3}$$

you try! FIND CRITICAL POINTS + MAX AND MINS

$$f'(x) = -\frac{2}{3}(-2x+10)^{-1/3} \cdot (-2)$$



$$f'(x) = \frac{4}{3(-2x+10)^{1/3}}$$

$x = 5$ WILL MAKE BOTTOM $= 0$

$$f'(0) = \frac{4}{3(0+10)^{1/3}} = \frac{4}{3(10)^{1/3}} = \text{positive \#}$$

$$f'(6) = \frac{4}{3(-2)^{1/3}} = \text{NEGATIVE \#}$$