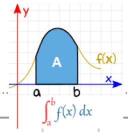
Note Title



U-Substitution

$\int$ integrals

5/26/2017

WHEN TWO FUNCTIONS ARE MULTIPLIED TOGETHER AND ONE FUNCTION APPEARS TO BE THE DERIVATIVE OF THE OTHER.

EX. 1

$$\int 3x^2 (x^3 - 5)^9 \, dx$$

(A) Let $u = x^3 - 5$

{ THE ONE THAT LOOKS LIKE IT WILL TURN INTO THEN 2nd FUNCTION WHEN IT'S DERIVATIVE IS PERFORMED. }

DO THE DERIVATIVE (B) $u = x^3 - 5$

$du = 3x^2 \, dx$

ⓒ SUBSTITUTE. $\int 3x^2 (u)^9 \frac{du}{3x^2}$

$\frac{du}{3x^2} = dx$

$\int u^9\, du$

$u = x^3 - 5$

$= \frac{u^{10}}{10} + C \Rightarrow \boxed{\frac{(x^3-5)^{10}}{10} + C}$

Ex. 2

$\int x \sqrt{5x^2 - 7}\, dx$

let $u = 5x^2 - 7$

$du = 10x\, dx$

$$\int x\sqrt{u}\,\frac{du}{10x}$$

$$dx = \frac{du}{10x}$$

$$\int \frac{u^{1/2}}{10} \rightarrow \frac{2}{3}\,\frac{u^{3/2}}{10} \rightarrow \frac{u^{3/2}}{15} + C$$

$$\rightarrow \boxed{\frac{(5x^2-7)^{3/2}}{15} + C}$$

Ex.3

$$\int \frac{1}{x(\ln x)^3}\,dx$$

let $u = \ln x$

$$du = \frac{1}{x}\,dx$$

$$dx = x\,du$$

$$\int \frac{1}{x\,u^3}\,x\,du \rightarrow \int \frac{1}{u^3}\,du$$

$$\int u^{-3}\,du$$

$$\frac{u^{-2}}{-2} + C$$

$$= \frac{(\ln x)^{-2}}{-2} + C$$

$$\boxed{= \frac{1}{-2(\ln x)^2} + C}$$

Ex. 4

$$\int \frac{e^{3t}}{e^{3t}+2}\,dt$$

let $u = e^{3t}+2$

$du = 3e^{3t}\,dt$

$$dt = \frac{du}{3e^{3t}}$$

$$\int \frac{e^{3t}}{u}\,\frac{du}{3e^{3t}}$$

$$\int \frac{1}{3u}\,du \rightarrow \frac{\ln u}{3}+C \rightarrow \frac{\ln(e^{3t}+2)}{3}+C$$

Ex. 5

SPECIAL TRICK MOVE → PUT x^5 INTO A FORM THAT LOOKS LIKE THE DERIVATIVE OF $1+x^2$

$$\int \sqrt{1+x^2}\;x^5$$

$$x^5 = x^4 \cdot \underline{\underline{x}}$$

$$\int \sqrt{u} \cdot x^4 \cdot x \cdot dx$$

let $u = 1 + x^2$

$du = 2x\,dx$

$dx = \dfrac{du}{2x}$

$$\int \sqrt{u} \cdot (u-1)^2 \frac{du}{2}$$

ALSO: $x^2 = u - 1$

$x^4 = (u-1)^2$

$$\int u^{1/2} \left(u^2 - 2u + 1\right) \frac{du}{2}$$

$$\int u^{5/2} - 2u^{3/2} + u^{1/2} \quad \frac{du}{2}$$

$$\frac{1}{2}\left[\frac{2u^{7/2}}{7} - 2 \cdot \frac{2u^{5/2}}{5} + \frac{2u^{3/2}}{3}\right]$$

$$\frac{(1+x^2)^{7/2}}{7} - \frac{2(1+x^2)^{5/2}}{5} + \frac{(1+x^2)^{3/2}}{3} + C$$

Ex: 6

$$\int (x+1)(x-2)^4\,dx$$

ANOTHER TRICK REQUIRED

Let $u = x-2$

$du = dx$ ✓

$$\int (x+1)(u)^4\,du$$

HAVE TO GET THIS IN TERMS OF u

$u = x-2$

$u+2 = x$

$u+3 = \boxed{x+1}$

ADD 1 TO BOTH SIDES!

$$\int (u+3)(u)^4\,du$$

$$\int u^{10} + 3u^9 \, du$$

$$\frac{1}{11}u^{11} + \frac{3}{10}u^{10} + C$$

$$= \frac{1}{11}(x-2)^{11} + \frac{3}{10}(x-2)^{10} + C$$

EX.7

$$\int (2x+3)\sqrt{2x-1}\,dx.$$

let $u = 2x - 1$

$du = 2\,dx$

$dx = \frac{du}{2}$

$$\int (2x+3)\frac{u^{1/2}}{2}\,du$$

GOTTA GET RID OF $(2x+3)$

$u = 2x-1$

$u+4 = 2x+3$

(ADD 4 TO BOTH SIDES)

$$\int (u+4)\frac{u^{1/2}}{2}\,du$$

$$\int \frac{u^{3/2}}{2} + 2u^{1/2}\,du$$

$$= \frac{u^{5/2}}{5} + \frac{4u^{3/2}}{3} + C$$

$$= \frac{(2x+1)^{5/2}}{5} \quad \frac{4(2x+1)^{3/2}}{3} + C \quad ✓$$

Ex. 8

DIVISION $\div$

$$\int \frac{x+5}{2x+3}\,dx$$

Let $u = 2x+3$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$\int \frac{x+5}{2u}\,du$$

PUT $(x+5)$ IN TERMS OF u

$u = 2x+3$

$\frac{u}{2} = x + \frac{3}{2}$

$\frac{u}{2} + \frac{7}{2} = x + 5$

$\frac{1}{2}(u+7) = x+5$

$$\int \frac{u+7}{2 \cdot 2u}\,du$$

$$\int \frac{u+7}{4u}\,du = \int \frac{1}{4} + \frac{7}{4u}\,du$$

$$= \frac{1}{4}u + \frac{7}{4}\ln u + C$$

$$= \frac{1}{4}(2x+3) + \frac{7}{4}\ln(2x+3) + C$$