Higher Order Derivatives

(Doing the derivative….*of a derivative*)

Let's recall what the meaning of a derivative is. A derivative is a function that will tell you the slope or **(RATE of CHANGE)** of the original function.

A good example of this is the **motion of an object** with respect to time:

position	$x = 6t^2 - t^3$	f(x)	Position vs. time function
velocity	$v = \frac{dx}{dt} = 12t - 3t^2$	**f'(x)**	first derivative
acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$	**f''(x)**	second derivative

We can see here that doing the derivative *(of a derivative)* has meaning. In fact doing a derivative more than once on a particular function is a common tool and can be useful in solving all sorts of problems. Doing the derivative *of a derivative* is called: **Higher Order Derivatives.**

Notation for **Higher Order Derivatives.**

1st: y', $f'(x)$, $\frac{dy}{dx}$,

2nd: y'', $f''(x)$, $\frac{d^2y}{dx^2}$,

3rd: y''', $f'''(x)$, $\frac{d^3y}{dx^3}$, Examples of Higher Order Derivatives:

1.

$$f(x) = 3x^2 + 4x$$

$$f\,'(x) = 6x + 4$$

$$f\,''(x) = 6$$

2.

$$f(x) = 4(x^2 - 1)^2$$

$$f\,'(x) = 8(x^2 - 1)(2x) = 16(x^3 - x)$$

$$f\,''(x) = 16(3x^2 - 1)$$

3. **Example**. Find the second derivative of the function

$$f(x)=x^2\cos(x)$$

Solution: By the product formula, we have

$$f'(x)=(2x)\cos(x)-x^2\sin(x)$$

Using the product formula twice more gives us the second derivative.

$$f''(x)=\left(2x\cos(x)\right)'-\left(x^2\sin(x)\right)'$$

$$=(2\cos(x)-2x\sin(x))-(2x)\sin(x)-x^2\cos(x)$$

$$=2\cos(x)-4x\sin(x))-x^2\cos(x)$$

4. $f(x) = 3x(x-1)^5$ Find $f\,'''(x)$. Note change in function.

$$f\,'(x) = 3x(5)(x-1)^4(1) + (x-1)^5(3)$$

$$= 3(x-1)^4\left[5x + x - 1\right] = 3(x-1)^4(6x-1)$$

$$f\,''(x) = 3\left[(x-1)^4(6) + (6x-1)(4)(x-1)^3\right]$$

$$= 3\cdot 2(x-1)^3\left[3(x-1) + 2(6x-1)\right]$$

$$= 6(x-1)^3\left[3x - 3 + 12x - 2\right]$$

$$= 6(x-1)^3(15x-5) = 30(x-1)^3(3x-1)$$

$$f\,'''(x) = 30\left[(x-1)^3(3) + (3x-1)(3)(x-1)^2\right]$$

$$= 90(x-1)^2\left[(x-1) + (3x-1)\right]$$

$$= 90(x-1)^2(4x-2) = 180(x-1)^2(2x-1)$$

Workbook Exercises:

Pg. 121 Ex.1   1,2,3

Pg. 122 Ex.2   2,3