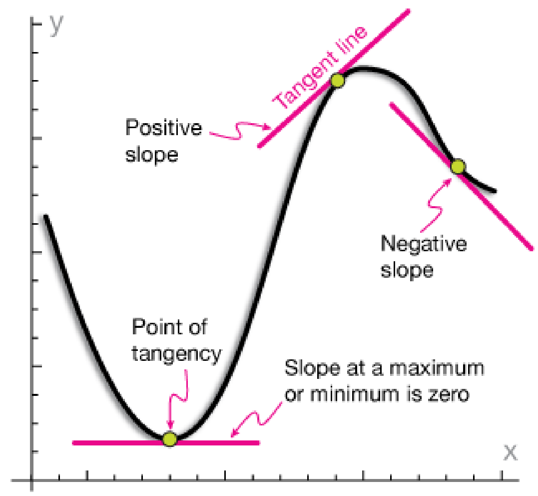
Differentiability

On many functions you can find the slope (rate of change) of that function on most if not all of its points.

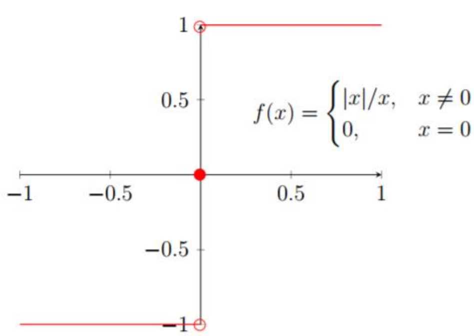
There are points on some function that you **CANNOT find the slope or derivative.**

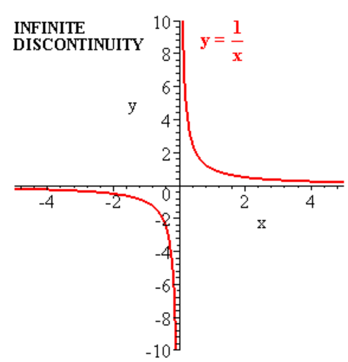
We call these point **non-differentiable**.



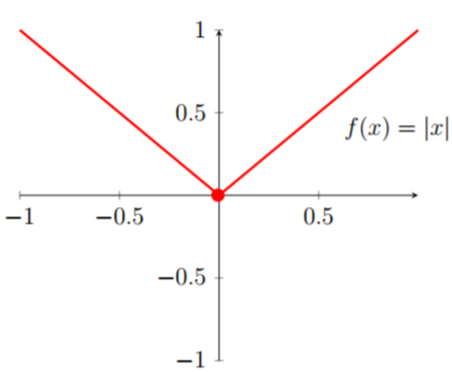
When will you have trouble finding the derivative at a particular point?

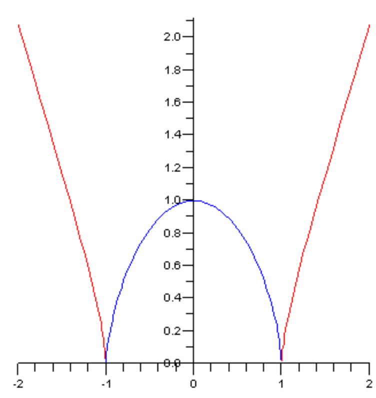
1. **If the function is not continuous at a particular point.**
   Then it is **not differentiable** at that point





2. **If the function has a "kink" (or cusp) in it.**
   Then it is **not differentiable** at that point

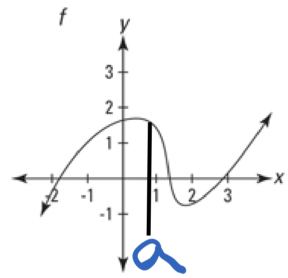


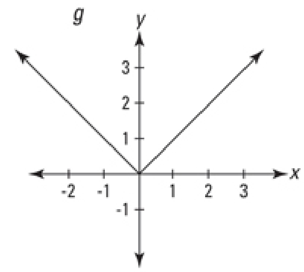


The conditions for **differentiability** at a given **point "a"** can be mathematically expressed in the following way:

1. The function must be **continuous** at **"a"**

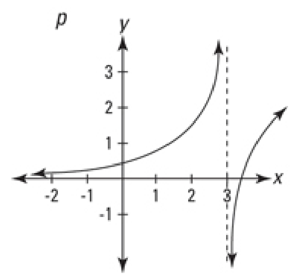
$$\lim_{x \to a^-} f(a) = \lim_{x \to a^+} f(a) = f(a)$$

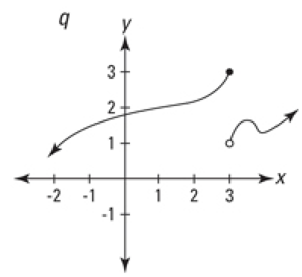




2. the **slope** of the function must be that same as you approach from each side of **"a"**

$$\lim_{x \to a^-} f'(a) = \lim_{x \to a^+} f'(a)$$





Example:

Determine if the following function is **differentiable** at x=3.

$$h(x) = \begin{cases} x^2 - 4x + 8, & x \le 3 \\ 2x - 1, & x > 3 \end{cases}$$

$h(3) = 5$

$\lim_{x \to 3^-} h(x) = 5$

$\lim_{x \to 3^+} h(x) = 5$

SUBTITUTION

$$h'(x) = \begin{cases} 2x - 4, & x \le 3 \\ 2, & x > 3 \end{cases}$$

$\lim_{x \to 3^-} h'(x) = 2$

$\lim_{x \to 3^+} h'(x) = 2$

$\lim_{x \to 3} h'(x) = 2$

SUBSTITUTION