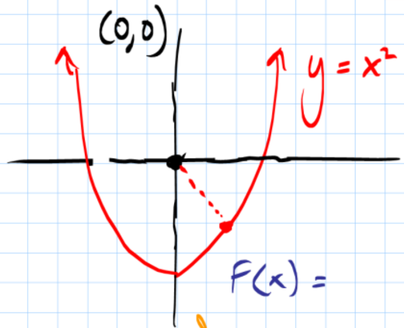
Example Set#2

MORE OPTIMIZATION EXAMPLES



$F(x) =$

CLASSIC CALCULUS

min distance

THE TRICK!

EQUATION FOR DISTANCE BETWEEN 2 POINTS!

(1) OBJECTIVE:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x-0)^2 + (y-0)^2}$$

(2) CONSTRAINT: HOW DOES x RELATE TO y

$$y = x^2 - 3$$

SO.....

$$d = \sqrt{x^2 + (x^2 - 3)^2}$$

$$d = \sqrt{x^2 + x^4 - 6x^2 + 9}$$

$$d = \sqrt{x^4 - 5x^2 + 9}$$

FIRST.... ANOTHER TRICK!

FINDING THE VALUE OF x THAT MINIMIZES THE VALUE $x^4 - 5x^2 + 9$

(FIND DERIVATIVE AND SET TO AND SET TO ZERO)

$f'(x) = 0$ (FOR MIN)

GIVES US THE SAME

x VALUE THAT MINIMIZES

$$\sqrt{x^4 - 5x^2 + 9}$$

SO....

$$\text{MIN DIST} = x^4 - 5x^2 + 9$$

$$\text{MIN DIST}' = 4x^3 - 10x$$

$$0 = 4x^3 - 10x$$

$$0 = 4x^2 - 10$$

$\sqrt{x}$ $\qquad$ x

MIN OF THIS SAME AS MIN FOR THIS

$x = \pm\sqrt{\frac{5}{2}}$ max/min

$x = 0$ max/min

SUB $x = \pm\sqrt{\frac{5}{2}}$ AND $x = 0$ INTO $d = \sqrt{x^4 - 5x^2 + 9}$

$$d = \sqrt{x^4 - 5x^2 + 9}$$

$$d = \sqrt{\left(\sqrt{\frac{5}{2}}\right)^4 - 5\left(\sqrt{\frac{5}{2}}\right)^2 + 9}$$

$$d = \sqrt{\frac{25}{4} - 5\left(\frac{5}{2}\right) + 9}$$

$$d = \sqrt{\frac{25}{4} - \frac{50}{4} + \frac{36}{4}}$$

$$\boxed{d = \sqrt{\frac{11}{4}}} = 1.66$$

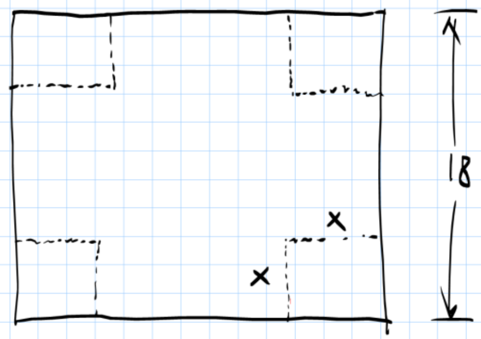
$$d = \sqrt{0^4 - 5(0)^2 + 9}$$

$$d = 3$$

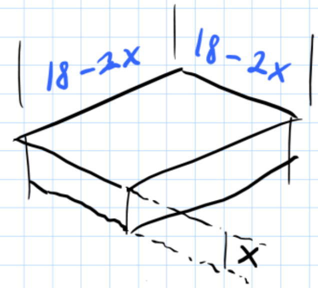
POINT

$$\left(\pm\sqrt{\frac{5}{2}}, y\right)$$

$$y = x^2 + 3$$

② 

SQUARE BASE



OBJECTIVE : VOLUME

$$V = \underset{\text{LENGTH}}{(18-2x)}\,\underset{\text{WIDTH}}{(18-2x)}\,\underset{\text{HEIGHT}}{(x)}$$

$$V = (324 - 36x - 36x + 4x^2)\,x$$

$$V = (324 - 72x + 4x^2)\,x$$

$$V = 324x - 72x^2 + 4x^3$$

$$V' = 324 - 144x + 12x^2$$

$$0 = 12(27 - 12x + x^2)$$

$$0 = (x-3)(x-9)$$

$x = 3, 9$ CHECK → 9 IS NO GOOD! CAN'T MAKE A BOX BY CUTTING IN 9 INCHES

MAKE $x = 3$

CUT IN 3 INCHES TO GET BOX WITH BIGGEST VOLUME