The Definition of an integral (the a Limit of a Riemann Sum)
Bonus exercise. (read the stuff below then answer the 5 questions on the **next page**)

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

1. To actually solve the integral as a limit you must be aware of the following summation formulas:

1. $\sum_{i=1}^{n} c = cn$

2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

2. Example using the **summation** formulas above:

Example 1 Using the formulas and properties from above determine the value of the following summation.

$$\sum_{i=1}^{100} (3-2i)^2$$

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The first thing that we need to do is square out the stuff being summed and then break up the summation using the properties as follows,

$$\begin{aligned}\sum_{i=1}^{100} (3-2i)^2 &= \sum_{i=1}^{100} 9 - 12i + 4i^2 \\ &= \sum_{i=1}^{100} 9 - \sum_{i=1}^{100} 12i + \sum_{i=1}^{100} 4i^2 \\ &= \sum_{i=1}^{100} 9 - 12\sum_{i=1}^{100} i + 4\sum_{i=1}^{100} i^2\end{aligned}$$

Now, using the formulas, this is easy to compute,

$$\begin{aligned}\sum_{i=1}^{100} (3-2i)^2 &= 9(100) - 12\left(\frac{100(101)}{2}\right) + 4\left(\frac{100(101)(201)}{6}\right) \\ &= 1293700\end{aligned}$$

Example:

Example of solving an Area problem using the limit of a Rieman Sum:
(using the **definition of an integral**….n → ∞)

evaluate the Riemann Sum for $f(x) = x^3$ from $x = 0$ to $x = 3$

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\left(\frac{3}{n}\right)$$

1. **Why is Δx = 3/n? What is the formula for Δx**

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right) \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^3$$

2. **Why is $f(x_i) = (3i/n)$? What is the formula for $f(x_i)$?**

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)^4 \sum_{i=1}^{n} i^3$$

$$= \lim_{n\to\infty} \left(\frac{3}{n}\right)^4 \left(\frac{n(n+1)}{2}\right)^2$$

3. **Explain how we got here from the previous line?**

$$= \left(\frac{3^4}{4}\right) \cdot \lim_{n\to\infty} \left(\frac{1}{n^4}\right)\left(\frac{n(n+1)}{1}\right)^2$$

$$= \left(\frac{3^4}{4}\right) \cdot \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^2$$

$$\int_0^3 x^3 dx = \left(\frac{3^4}{4}\right)$$

$$\int_0^3 x^3 dx = \frac{81}{4}$$

4. **Use an online integral calculator (or perform this definite integral yourself) to confirm that the AREA under x^3 from 0 to 3 is …..81/4.**

5. **Use the definition of an integral (as a limit of a summation) to find the following:**

$$\int_0^1 3x^2 dx$$

Answer: 1 unit^2