Calculus 12

Limits Self-Exam

June 2020

You can use this as more of a review then an exam. Don't be afraid to have your notes and materials out to assist you. Solutions at end of Exam.

Evaluate each limit.

1. $\lim_{x\to 3}\left(x^2+2\right)$

2. $\lim_{x\to -3}\dfrac{(x+3)(x-4)}{x^2+4x+3}$

3. $\lim_{x\to 25}\dfrac{\sqrt{x}-5}{x-25}$

4. $\lim_{x\to -2}\dfrac{x-4}{x^2-2x-8}$

5. $\lim_{x\to -3}\dfrac{x^2+2x-3}{x^2+7x+12}$

6. $\lim_{x\to -2}\dfrac{x^3+8}{x+2}$

7. $\lim_{x\to 5}\dfrac{x-5}{|x-5|}$

8. $\lim_{x\to 8}\dfrac{1}{x-8}$

9. $\lim_{x\to \infty} e^{-x}$

10. $\lim_{x\to \infty}\dfrac{2x+5x^2-6x^3}{15x^4+100x^2}$

11. $\lim_{x\to \infty}\dfrac{2x+3}{5-3x}$

12. $\lim_{x\to 0^+}\ln x$

13. $\lim_{x\to 2^+}\dfrac{|x-2|}{x-2}$

14. $\lim_{x\to 0}\dfrac{\sin 5x}{x}$

15. $\lim_{x\to 0}\dfrac{\cos x-1}{x}$

16. $\lim_{x\to 0^-}\dfrac{1}{x}$

17. $\lim_{x\to \infty}\arctan x$

18. $\lim_{x\to \frac{\pi}{2}}\sin x$

19. $\lim_{x\to \infty}\cos x$

20. $\lim_{x\to 1^+}[x]$

21. $\lim_{x\to 0}\dfrac{\tan 3x}{5x}$

22. $\lim_{n\to 0}\left(5^n+\dfrac{1}{5^n}\right)$

23. $\lim_{n\to 0}\dfrac{(1+n)^2-1}{n}$

24. $\lim_{n\to \infty}\dfrac{(n-2)(n+1)}{n^2}$

25. $\lim_{x\to \infty}\dfrac{2x-5}{x}$

26. $\lim_{x\to 0}\dfrac{\sin^2 x}{x}$

27. $\lim_{x\to 0}\dfrac{\sin 3x}{5x}$

28. $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}$

29. $\lim_{x\to 2} f\left[g(x)\right]$, **where** $f(x)=3x-2$ **and** $g(x)=x^2-1$

30. $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{\sin x - \cos x}{\cos 2x}$

31. $\lim\limits_{t\to 0} \dfrac{\tan 6t}{\sin 2t}$

32. $\lim\limits_{x\to\infty} x - \sqrt{x^2+7}$

33. $\lim\limits_{x\to\infty} \left(\dfrac{1}{2}\right)^x$

34. $\lim\limits_{x\to\infty} xe^{-x} + 5e^{-x}$

35. $\lim\limits_{x\to\frac{\pi}{2}^+} \dfrac{1}{2^{\tan(x)}}$

36. $\lim\limits_{x\to\infty} \ln x$

37. $\lim\limits_{x\to 0^+} \ln x$

38. $\lim\limits_{x\to\infty} \sin x$

39. $\lim\limits_{x\to 0} \dfrac{1}{x^2}$

40. $\lim\limits_{x\to\infty} \dfrac{x^2 \cos\left(\dfrac{1}{x}\right)}{2x^2-1}$

41. $\lim\limits_{x\to-\infty} \dfrac{2x-7}{\sqrt{3x^2+1}}$

42. $\lim\limits_{x\to-\infty} \dfrac{3^x - 3^{-x}}{3^x + 3^{-x}}$

43. $\lim\limits_{x\to\infty} \dfrac{4x^{\frac{5}{2}} - 7x}{x^2+10x}$

44. $\lim\limits_{x\to\infty} \dfrac{\cos(\pi x)}{x}$

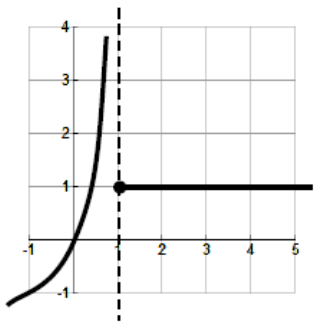
45-50: For each of the following determine: **a)** $\lim\limits_{x\to 1^-} f(x)$ **b)** $\lim\limits_{x\to 1^+} f(x)$ **c)** $\lim\limits_{x\to 1} f(x)$

45. $f(x) = \begin{cases} x^2-1 & x<1 \\ 4-x & x\geq 1 \end{cases}$

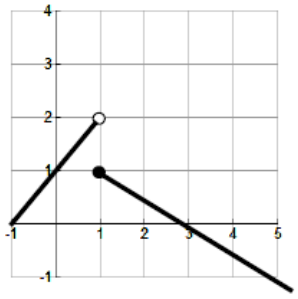
46. $f(x) = \begin{cases} 3x-1 & x\leq 1 \\ 3-x & x>1 \end{cases}$

47. $f(x) = \begin{cases} -x^2 & x<1 \\ 2 & x=1 \\ x-2 & x>1 \end{cases}$

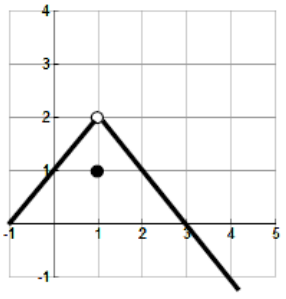
48.



49.



50.



AP Calculus AB
Limit Worksheet Ans

1. 11	**2.** $\frac{7}{2}$	**3.** $\frac{1}{10}$	**4.** *DNE*	**5.** -4
6. 12	**7.** *DNE*	**8.** *DNE*	**9.** 0	**10.** 0
11. $-\frac{2}{3}$	**12.** $-\infty$	**13.** 1	**14.** 5	**15.** 0
16. $-\infty$	**17.** $\frac{\pi}{2}$	**18.** 1	**19.** *DNE*	**20.** 1
21. $\frac{3}{5}$	**22.** 2	**23.** 2	**24.** 1	**25.** 2
26. 0	**27.** $\frac{3}{5}$	**28.** $\frac{1}{2}$	**29.** 7	**30.** $-\frac{\sqrt{2}}{2}$
31. 3	**32.** 0	**33.** 0	**34.** 0	**35.** $+\infty$
36. $+\infty$	**37.** $-\infty$	**38.** *DNE*	**39.** $+\infty$	**40.** $\frac{1}{2}$
41. $-\frac{2\sqrt{3}}{3}$	**42.** 1	**43.** $+\infty$	**44.** 0	**45.** $0, 3, DNE$
46. $2, 2, 2$	**47.** $-1, -1, -1$	**48.** $+\infty, 1, DNE$	**49.** $2, 1, DNE$	**50.** $2, 2, 2$

Limits, Continuity and Piecewise Functions

Evaluate each limit if it exists.

1. $\lim_{x\to 2} f(x)=\begin{cases} x+1 \text{ if } x<2 \\ x^2-1 \text{ if } x\geq 2 \end{cases}$
2. $\lim_{x\to 1} f(x)=\begin{cases} 2-x \text{ if } x<1 \\ x+4 \text{ if } x\geq 1 \end{cases}$
3. $\lim_{x\to 5}\dfrac{x-5}{x-5}$
4. $\lim_{x\to 3}\dfrac{x^2-9}{x^2+9}$
5. $\lim_{x\to 2}\dfrac{x^3-8}{x^2-4}$
6. $\lim_{x\to\infty}\dfrac{2-x+x^3}{4x^3-5x+100}$
7. $\lim_{x\to\infty}\dfrac{7x^5+100x^6}{x^3-11x^5}$
8. $\lim_{x\to\infty}\dfrac{\sqrt{x^2+5x+12}}{x-3}$
9. $\lim_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}$
10. $\lim_{x\to -1^+}\dfrac{|x+1|}{x+1}$
11. $\lim_{x\to -\infty} xe^x$
12. $\lim_{x\to 0}\dfrac{\tan 8x}{\sin 3x}$
13. Find the value of k such that f is continuous at $x=1$

$$f(x)=\begin{cases} -(x-1)^2+1 & \text{if } x<1 \\ k & \text{if } x=1 \\ (x-1)^2+1 & \text{if } x>1 \end{cases}$$

14. Find the value of k such that f is continuous at $x=2$

$$f(x)=\begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x\neq 2 \\ k & \text{if } x=2 \end{cases}$$

15. Find the value of k such that f is continuous at $x=0$

$$f(x)=\begin{cases} \dfrac{2x^3-x}{3x} & \text{if } x\neq 0 \\ f(0)=k \end{cases}$$

16. Is the function $f(x)$ continuous? If not, state where and what type of discontinuity it contains.

$$f(x)=\begin{cases} \dfrac{3x(x-1)}{x^2-3x+2} & \text{if } x\neq 1,2 \\ f(1)=-3 & \\ 4 & \text{if } x=2 \end{cases}$$

17. If $f(x)=\begin{cases} 2x^2+3 & \text{if } x\geq 1 \\ g(x) & \text{if } x<1 \end{cases}$ then f will be continuous at $x=1$, if $g(x)=?$
    a. x
    b. $\cos(x+4)$
    c. $6-x$
    d. x^2+2
    e. $2x^2-3$
18. Given $f(x)=\begin{cases} x^2-4 & \text{if } x\neq 1 \\ 2 & \text{if } x=1 \end{cases}$, determine which statement(s) must be true.
    a. $\lim_{x\to 1} f(x)=\text{exists}$
    b. $f(1)$ *exists*
    c. f is continuous at $x=1$

Limits, Continuity and Piecewise Functions

Evaluate each limit if it exists.

1. $\lim_{x\to 2} f(x) = \begin{cases} x+1 \text{ if } x<2 \\ x^2-1 \text{ if } x\geq 2 \end{cases} = 3$

2. $\lim_{x\to 1} f(x) = \begin{cases} 2-x \text{ if } x<1 \\ x+4 \text{ if } x\geq 1 \end{cases} = DNE$

3. $\lim_{x\to 5} \frac{x-5}{x-5} = 1$

4. $\lim_{x\to 3} \frac{x^2-9}{x^2+9} = 0$

5. $\lim_{x\to 2} \frac{x^3-8}{x^2-4} = 3$

6. $\lim_{x\to\infty} \frac{2-x+x^3}{4x^3-5x+100} = \frac{1}{4}$

7. $\lim_{x\to\infty} \frac{7x^5+100x^6}{x^2-11x^5} = +\infty$

8. $\lim_{x\to\infty} \frac{\sqrt{x^2+5x+12}}{x-3} = 1$

9. $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \frac{1}{6}$

10. $\lim_{x\to -1^+} \frac{|x+1|}{x+1} = 1$

11. $\lim_{x\to -\infty} xe^x = 0$

12. $\lim_{x\to 0} \frac{\tan 8x}{\sin 3x} = \frac{8}{3}$

13. Find the value of k such that f is continuous at $x=1$

$$f(x) = \begin{cases} -(x-1)^2+1 & \text{if } x<1 \\ k & \text{if } x=1 \\ (x-1)^2+1 & \text{if } x>1 \end{cases} \quad k=1$$

14. Find the value of k such that f is continuous at $x=2$

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x\neq 2 \\ k & \text{if } x=2 \end{cases} \quad k=4$$

15. Find the value of k such that f is continuous at $x=0$

$$f(x) = \begin{cases} \frac{2x^2-x}{3x} & \text{if } x\neq 0 \\ f(0)=k \end{cases} \quad k=-\frac{1}{3}$$

16. Is the function $f(x)$ continuous? If not, state where and what type of discontinuity it contain

$$f(x) = \begin{cases} \frac{3x(x-1)}{x^2-3x+2} & \text{if } x\neq 1,2 \\ f(1)=-3 \\ 4 & \text{if } x=2 \end{cases}$$

Discontinuous at $x=2$, infinite

17. If $f(x) = \begin{cases} 2x^2+3 & \text{if } x\geq 1 \\ g(x) & \text{if } x<1 \end{cases}$ then f will be continuous at $x=1$, *if* $g(x)=?$

   a. x
   b. $\cos(x+4)$
   c. $6-x$
   d. x^2+2
   e. $2x^2-3$

18. Given $f(x) = \begin{cases} x^2-4 & \text{if } x\neq 1 \\ 2 & \text{if } x=1 \end{cases}$, determine which statement(s) must be true.

   a. $\lim_{x\to 1} f(x) = exists$
   b. $f(1)$ *exists*
   c. f is continuous at $x=1$