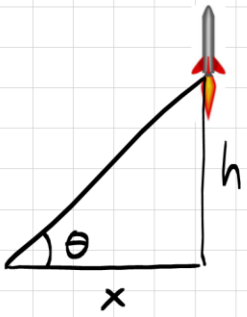
RELATED RATES EXAMPLES #2



FIND $\frac{d\theta}{dt}$

$\frac{dh}{dt} = 2.5 \text{ ft/s}$

$x = 15$

$$\tan\theta = \frac{h}{15}$$

DERIVATIVE WITH RESPECT TO t!

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{15}\frac{dh}{dt}$$

IMPORTANT!

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{15}\frac{dh}{dt}$$

$$\left(\frac{1}{\cos\theta}\right)^2 \frac{d\theta}{dt} = \frac{1}{15}(2.5)$$

$$2\frac{d\theta}{dt} = 0.1666$$

$$\frac{d\theta}{dt} = 0.083 \text{ RADS/S}$$

$$\frac{dh}{dt} = 2.5 \text{ ft/s}$$

$\frac{d\theta}{dt} = ?$

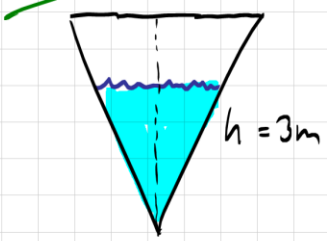
$\theta = 0.785$ RADIANS!

$$\tan\theta = \frac{15\text{ft}}{15\text{ft}}$$

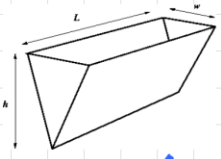
$\tan\theta = 1$

$\theta = 0.785$

Ex 2



RELATE VOLUME TO HEIGHT



$V = \frac{1}{2} wh(L)$ TRUE!

$V = \frac{1}{2} wh(15)$ L CONSTANT!

TRICK:

AS WATER RISES, w AND h WILL ALWAYS HAVE THE SAME RATIO.

$\frac{w}{h} = \frac{4}{3}$ $\quad w = \frac{4}{3}h$

$V = \frac{1}{2}\left(\frac{4}{3}h\right)h(15)$

$\boxed{V = 10h^2}$

$V = 10h^2$ NOW DERIVATIVE WITH RESPECT TO TIME (t)

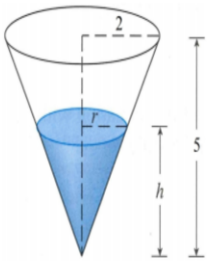
$$\frac{dV}{dt} = 20h\frac{dh}{dt}$$

$$2.5\ m^3/min = 20(2)\frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.0625\ m/min$$

NOTE:

SAME IDEA WORKS FOR FILLING TRIANGULAR CONE

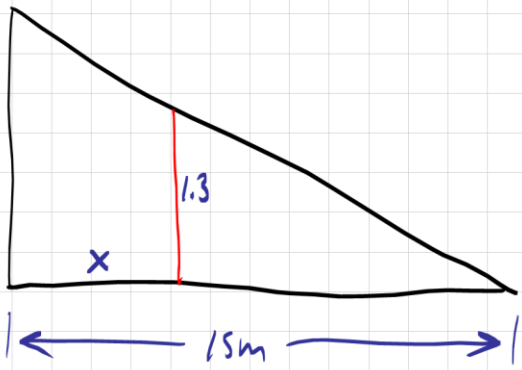


$V = \frac{1}{3}\pi r^2 h$

RATIO OF $h \to r$ $\quad \frac{r}{h} = \frac{2}{5}$

(#3) SIMILAR TRIANGLES!

A)



B) $\frac{y}{15} = \frac{1.3}{15 - x}$

$(1.3)(15) = y(15 - x)$

$19.5 = 15y - xy$ PRODUCT RULE

C)

$0 = 15\frac{dy}{dt} - \left[\frac{dx}{dt}y + x\frac{dy}{dt}\right]$

$0 = 15\frac{dy}{dt} - \left[(-3.1)(1.95) + 5\frac{dy}{dt}\right]$

$0 = 15\frac{dy}{dt} - \left[-6.045 + 5\frac{dy}{dt}\right]$

$0 = 15\frac{dy}{dt} + 6.045 - 5\frac{dy}{dt}$

D) SOLVE FOR y

$x = 5$

$\frac{y}{15} = \frac{1.3}{15 - 5}$

$y = 1.95$

$-6.045 = 10\frac{dy}{dt}$

$\frac{dy}{dt} = -0.6\ m/s$