Integration Problem Set

1. Evaluate $\sum_{k=1}^{4} k^k$.

2. Express $-1 + 0 + \frac{1}{3} + \frac{2}{4} + \frac{3}{5}$ using $\sum$ notation. “i” will start at negative 1 in this case

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left[\sum_{i=1}^{n} f(x_i)\Delta x\right]$$

Use the formula above to answer the following 2 multiple choice questions:

3. What type of quantity does the above formula represent?

a) slope of *f(x)* at a
b) the area of a rectangle
c) the area of x number of rectangles
d) the area under *f(x)* form a to b

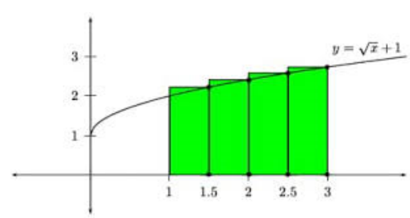
4. What does $f(x_i)$ equal in the formula above?

a) $\Delta x + n$
b) $a + i\cdot\Delta x$
c) $(b-a)/n$
d) the area of one rectangle

5. Given the formula and image to the right
What is n?

$$\sum_{i=1}^{4} f(x_i)\cdot\Delta x$$

a) $a + i\cdot\Delta x$
b) Δx
c) 4
d) the number of rectangles
e) both c and d

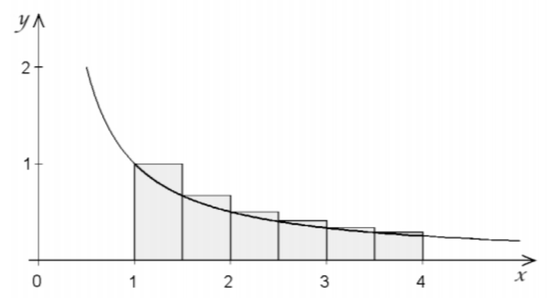


6. What is the value of Δx in the diagram shown to the right?

a) 4
b) 1
c) b-a
d) 0.5

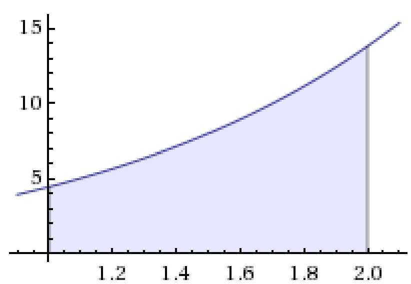
7. let $f(x) = \dfrac{1}{x}$ on the interval $[1, 4]$.

Estimate the area under *f(x)* using 6 rectangles



8. Find the approx area below $y = 2e^x - 1$ from 1 to 2. Use $n = 4$.

(answer: 9.56)



QUESTION #9

2. $\int(-6x^3+9x^2+4x-3)dx$

4. $\int\left(\frac{8}{x}-\frac{5}{x^2}+\frac{6}{x^3}\right)dx$

6. $\int(12x^{\frac{3}{4}}-9x^{\frac{5}{3}})dx$

8. $\int\frac{1}{x\sqrt{x}}dx$

10. $\int(2t^2-1)^2\,dt$

12. $\int d\theta$

14. $\int 5\cos(\theta)d\theta$

16. $\int 12\cos(4\theta)d\theta$

18. $\int 4\sin\left(\frac{x}{3}\right)dx$

20. $\int 9e^{\frac{x}{4}}dx$

22. $\int -13e^{6t}\,dt$

Question#10

II. Evaluate the following definite integrals.

1. $\int_1^4 (5x^2 - 8x + 5)dx$

2. $\int_1^9 (x^{\frac{3}{2}} + 2x + 3)dx$

3. $\int_4^9 (\sqrt{x} + \frac{1}{3\sqrt{x}})dx$

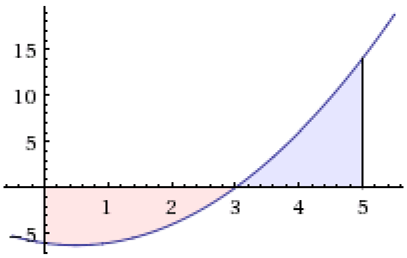
4. $\int_1^4 \frac{5}{x^3} dx$

5. $\int_{-1}^2 (1+3t)t^2 \, dt$

6. $\int_{-2}^1 (2t^2 - 1)^2 \, dt$

Finding Areas

11. Find both the *net* and *gross* area of the area bound by the function below $y = x^2 - x - 6$.

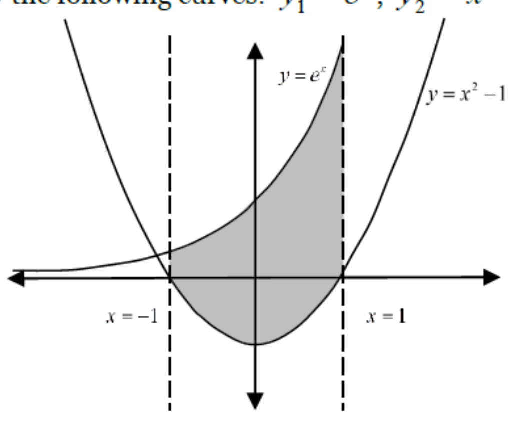


12. Find the both the *net* and *gross* areas bound by the x-axis and the function below:

$f(x) = x^3 + 2x^2 - 3x.$ Use a graphing utility or the internet to first create a sketch.

13. Find the area of the region enclosed by the following curves: $y_1 = e^x$, $y_2 = x^2 - 1$, $x = -1$ and $x = 1$.

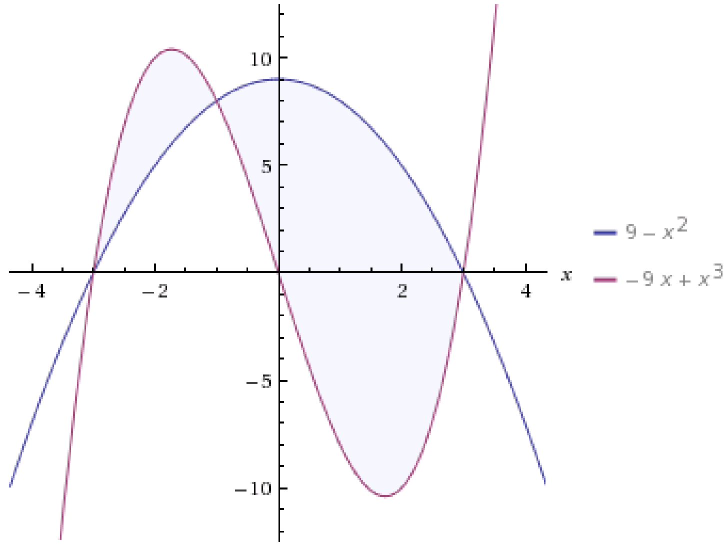
As always, we will first draw a sketch.



14.

a) Have a peek at the graph shown to the right representing the area bound by:

$9 - x^2$ and $x^3 - 9x$



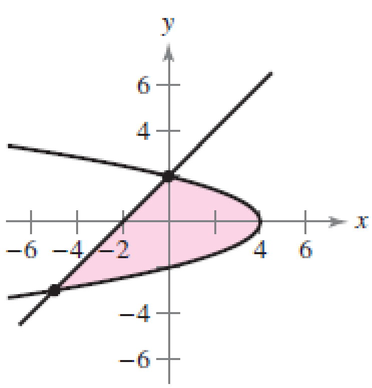
b) Determine all the *boundaries* and *intersections* needed to find this area (*without relying on the graph).*

c) Find the area bound by the two functions.

15. Find the area bound by the functions show below. Find intercepts first – show your work.

$x = 4 - y^2$

$x = y - 2$



16.

Find the area of the region bounded by $y^2 + 1$, $x = 0$, $y = 1$, $y = 2$. *(of course draw a sketch first – show all your work)*

Answers:

1. 288
2. $\sum_{i=-1}^{5} \frac{1}{i}$
3. d
4. b
5. e
6. d
7. 1.21
8. ***9.56***

9. **Watch the numbering please! Even ones only**

2. $\int(-6x^3+9x^2+4x-3)dx = \boxed{\frac{-3x^4}{2}+3x^3+2x^2-3x+C}$

3. $\int(x^{\frac{3}{2}}+2x+3)dx = \boxed{\frac{2x^{\frac{5}{2}}}{5}+x^2+3x+C}$

4. $\int\left(\frac{8}{x}-\frac{5}{x^2}+\frac{6}{x^3}\right)dx = \int\left(\frac{8}{x}-5x^{-2}+6x^{-3}\right)dx$
$= 8Ln(x)-\frac{5x^{-1}}{-1}+\frac{6x^{-2}}{-2} = \boxed{8Ln(x)+\frac{5}{x}-\frac{3}{x^2}+C}$

5. $\int(\sqrt{x}+\frac{1}{3\sqrt{x}})dx = \int\left(x^{\frac{1}{2}}+\frac{1}{3}x^{-\frac{1}{2}}\right)dx$
$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{1}{3}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \boxed{\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{3}x^{\frac{1}{2}}+C}$

6. $\int(12x^{\frac{3}{4}}-9x^{\frac{5}{3}})dx = \boxed{\frac{48x^{\frac{7}{4}}}{7}-\frac{27x^{\frac{8}{3}}}{8}+c}$

7. $\int\frac{x^2+4}{x^2}dx = \int 1+4x^{-2}\ \ dx = \boxed{x-\frac{4}{x}+C}$

8. $\int\frac{1}{x\sqrt{x}}dx = \int x^{-\frac{3}{2}}dx = \boxed{-\frac{2}{\sqrt{x}}+C}$

12. $\int d\theta = \boxed{\theta+C}$

14. $\int 5\cos(\theta)d\theta = \boxed{5\sin(\theta)+C}$

16. $\int 12\cos(4\theta)d\theta = \boxed{3\sin\ 4\theta\ +C}$

18. $\int 4\sin\left(\frac{x}{3}\right)dx = \boxed{-12\cos\left(\frac{x}{3}\right)+C}$

20. $\int 9e^{\frac{x}{4}}dx = \boxed{36e^{\frac{x}{4}}+C}$

22. $\int -13e^{6t}dt = \boxed{-\frac{13e^{6t}}{6}+C}$

10.

1. $\int_1^4(5x^2-8x+5)dx = \left(\frac{5x^3}{3}-4x^2+5x\right)\Big|_1^4 = \frac{188}{3}-\frac{8}{3} = \boxed{60}$

2. $\int_1^9(x^{\frac{3}{2}}+2x+3)dx = \left(\frac{2x^{\frac{5}{2}}}{5}+x^2+3x\right)\Big|_1^9 = \frac{1026}{5}-\frac{22}{5} = \boxed{\frac{1001}{5}=200.2}$

3. $\int_4^9(\sqrt{x}+\frac{1}{3\sqrt{x}})dx = \left(\frac{2}{3}x^{\frac{3}{2}}+\frac{2}{3}x^{\frac{1}{2}}\right)\Big|_4^9 = 20-\frac{20}{3} = \boxed{\frac{40}{3}=13.333}$

4. $\int_1^4\frac{5}{x^3}dx = -\frac{5}{2x^2}\Big|_1^4 = -\frac{5}{32}+\frac{5}{2} = \boxed{\frac{75}{32}=2.344}$

5. $\int_{-1}^2(1+3t)t^2\,dt = \left(\frac{t^3}{3}+\frac{3t^4}{4}\right)\Big|_{-1}^2 = \frac{44}{3}-\frac{5}{12} = \boxed{\frac{57}{4}=14.25}$

6. $\int_{-2}^1(2t^2-1)^2\,dt = \left(\frac{4t^5}{5}-\frac{4t^3}{3}+t\right)\Big|_{-2}^1 = \frac{7}{15}+\frac{254}{15} = \boxed{\frac{87}{5}=17.4}$

11. [*Answer: Net: – 0.833 Gross: 26.166*]
12. [Answer: Net: 10.667, Gross: 11.8333]
13. [*Answer: 3.68*]
14. [*answer: 49.333*]
15. [*answer: 20.8333*]

16. Answer: 3.33