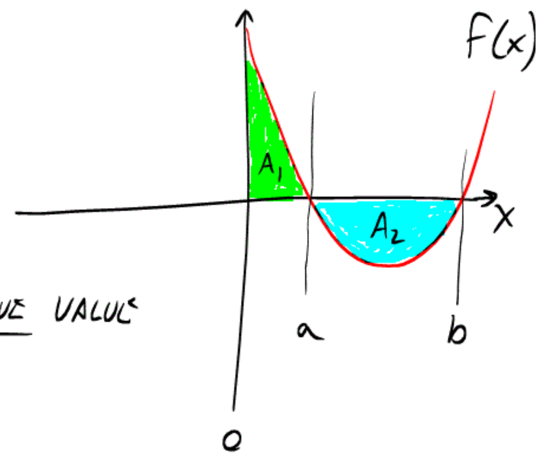
WHEN AREA IS BELOW THE X-AXIS



A_1 is positive

A_2 is negative.

$\int_0^a f(x)$ = will be a <u>positive</u> value

$\int_a^b f(x)$ = will be a <u>negative</u> value

Net Area:

Any Area below the x-axis is negative. What this really means depends on the application. If we were to integrate from **0** to **b** in the example above, this would give us the **Net Area** and the integral would automatically subtract A_2 from A_1 to give a **Net Area** (no special attention is required).

Gross Area:

However, if we want to find out **the total physical area of the shaded areas above** we would have to *ignore* that area below the graph **is negative.**
We would call this area the **Gross Area**.
(see examples on next page)

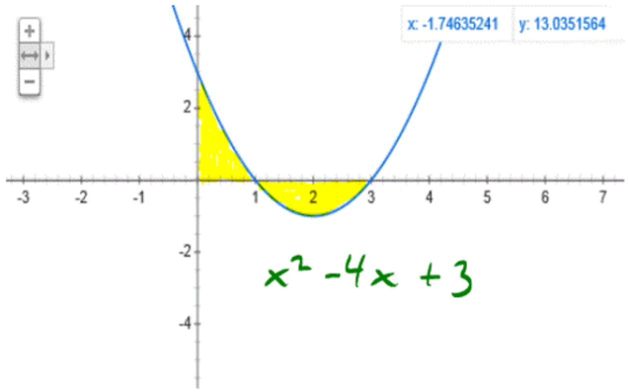
<u>Gross</u> Area $= A_1 + A_2 = |A_1| + |A_2|$

<u>Net</u> Area $= A_1 - |A_2|$ ⟸ Just integrate as usual

Above graph Below graph

EXAMPLE

FIND BOTH THE NET AN GROSS AREA OF $x^2 - 4x + 3$ FROM $0 \to 3$



NET AREA $\to$ JUST INTEGRATE FROM $0 \to 3$ NORMALLY

(THE INTEGRAL TAKES CARE OF + or − AREAS)

$$\int_0^3 x^2 - 4x + 3 = \left[\frac{1}{3}x^3 - 2x^2 + 3x \right]_0^3$$

$$= \frac{1}{3}(3)^3 - 2(3)^2 + 3(3) - \frac{1}{3}(0) - 2(0)^2 - 3(0)$$

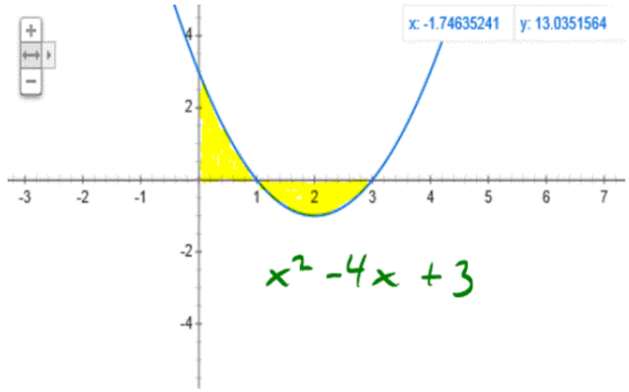
$$= 9 - 18 + 9 - 0$$

$$= 0$$ NET AREA

Gross Area next page….

EXAMPLE

FIND BOTH THE NET AN GROSS AREA OF $x^2 - 4x + 3$ FROM $0 \to 3$



GROSS AREA

FIND AREA ABOVE AND BELOW X-AXIS AND ADD TOGETHER.

$$\int_0^1 x^2 - 4x + 3 \quad + \quad \int_1^3 x^2 - 4x + 3$$

ABSOLUTE VALUE OF

$$\tfrac{1}{3}(1)^3 - 2(1) + 3(1) \quad + \quad \left[\, 0 - \left(+ \tfrac{1}{3}(1)^3 - 2(1) + 3(1) \right) \right]$$

$$= \frac{1}{3} + 1 \quad +$$

ABSOLUTE VALUE!

$$= \frac{4}{3} \quad + \quad \left[- \frac{4}{3} \right]$$

$$= \frac{8}{3}$$ GROSS AREA.