

## DIFFERENTIAL EQUATIONS - GROWTH AND DECAY PROBLEMS

Note Title

6/13/2017

IN MANY CASES THE  
RATE OF CHANGE OF A QUANTITY  
IS DIRECTLY PROPORTIONAL  
TO THE QUANTITY ITSELF

### PROBLEMS INVOLVING:

- POPULATION
- FINANCE (INTEREST)
- NUCLEAR DECAY
- HEATING / COOLING



$$\frac{dy}{dt} = ky$$

$k$  - CONSTANT RELATED TO HOW RATE CHANGES WITH QUANTITY

$y$  - QUANTITY  
 $t$  - time

SOLVE:

$$\frac{1}{y} dy = k dt$$

$$\int \frac{1}{y} dy = \int k dt$$

NOTE:

$k > 0$  GROWTH  
 $k < 0$  DECAY

$$\ln y = kt + c$$

"e" BOTH SIDES

$$e^{\ln y} = e^{kt+c} \rightarrow e^{kt} \cdot e^c = Ae^{kt}$$

$$y = Ae^{kt}$$

TRUE!

y - QUANTITY

t - time

k - RATE FACTOR

A - FACTOR RELATED TO  
CONDITIONS OF THE PROBLEM.

Example #1

$$y = Ae^{kt}$$

y - POPULATION      A - (HAVE TO FIND OUT)  
t - time  
k - GROWTH RATE

$$P = A e^{0.038t}$$

(FIND POPULATION WHEN  $t = 5.2$  years)  
(GOTTA FIND A FIRST)

$$1543 = A e^{0.038(0)} \leftarrow @ t = 0 P = 1543$$

$$1543 = A(1)$$

$$A = 1543$$

$$\underline{P = 1543 e^{0.038(5.2)}}$$

$$P = 1543 e^{0.1976}$$

$$P = 1543 (1.218)$$

$$P = 1880 (\text{people})$$

- 1) For a period of time, an island's population grows at a rate proportional to its population. If the growth rate is 3.8% per year and the current population is 1543, what will the population be 5.2 years from now?

Ex 4

$$y = Ae^{kt}$$

$$1077 = Ae^{0.031t}$$

$$1077 = Ae^{0.031(0)} \leftarrow y = 1077, t = 0$$

$$A = 1077$$

$$y = 1077 e^{0.031t}$$

$$1486.73 = 1077 e^{0.031t}$$

$$\frac{1486.73}{1077} = e^{0.031t}$$

$$1.38 = e^{0.031t}$$

y - BALANCE

t - time

k - 3.1% (0.031)

A - FIND OUT

- 4) A savings account balance is compounded continuously. If the interest rate is 3.1% per year and the current balance is \$1077.00, in how many years will the balance reach \$1486.73?

"ln" BOTH SIDES

$$\ln 1.38 = 0.031t$$

$$0.322 = 0.031t$$

$$t = 10.4 \text{ years}$$

Ex. 6

$$y = Ae^{kt}$$

$y$  - POPULATION  
 $t$  - TIME

$K$  - RATE FACTOR  
 $A$  - FIND OUT.

@  $t = 0$

$$y = Ae^{k(0)}$$

$$y = A$$

$A$  equals INITIAL POPULATION of "y"

Now DOUBLE "y" IN 20.4 min TO FIND  $K$

$$y = Ae^{kt}$$

- 6) During the exponential phase, E. coli bacteria in a culture increase in number at a rate proportional to the current population. If the population doubles in 20.4 minutes, in how many minutes will the population triple?

$$2y = ye^{k(20.4)}$$

TRUE!

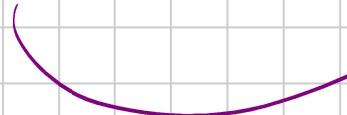
$$2 = e^{k(20.4)}$$

"ln" BOTH SIDES

$$\ln 2 = k(20.4)$$

$$k = \frac{\ln 2}{20.4}$$

$$k = 0.0339$$



$$y = Ae^{0.0339t}$$

(POPULATION  $\times 3$ )

$$3y = ye^{0.0339t}$$

$$3 = e^{0.0339t}$$

$$\ln 3 = 0.0339t$$

$$t = 32.4 \text{ min}$$