DIFFERENTIAL EQUATIONS - GROWTH AND DECAY PROBLEMS

Note Title 6/13/2017

IN MANY CASES THE RATE OF CHANGE OF A QUANTITY IS DIRECTLY PROPORTIONAL TO THE QUANTITY ITSELF

PROBLEMS INVOLVING:

- POPULATION
- FINANCE (INTEREST)
- NUCLEAR DECAY
- HEATING / COOLING

$$\frac{dy}{dt} = ky$$

k - CONSTANT RELATED TO HOW RATE CHANGES WITH QUANTITY

y - QUANTITY

t - TIME

SOLVE:

$$\frac{1}{y}\,dy = k\,dt$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

NOTE:

$k > 0$ GROWTH

$k < 0$ DECAY

$\ln y = kt + c$ "e" BOTH SIDES

$e^{\ln y} = e^{kt+c}$ $\rightarrow e^{kt} \cdot e^{c} = Ae^{kt}$

$$y = Ae^{kt}$$

TRUE!

y - QUANTITY
t - time
k - RATE FACTOR
A - FACTOR RELATED TO CONDITIONS OF THE PROBLEM.

example #1

$$y = Ae^{kt}$$

y - POPULATION A - (HAVE TO FIND OUT)
t - time
k - GROWTH RATE

$$P = Ae^{0.038t}$$

(FIND POPULATION WHEN $t = 5.2$ years)
(GOTTA FIND A FIRST)

$$1543 = Ae^{0.038(0)}$$

@ $t = 0$ $P = 1543$

$$1543 = A(1)$$

$$A = 1543$$

1) For a period of time, an island's population grows at a rate proportional to its population. If the growth rate is 3.8% per year and the current population is 1543, what will the population be 5.2 years from now?

$$P = 1543e^{0.038(5.2)}$$

$$P = 1543e^{0.1976}$$

$$P = 1543(1.218)$$

$$P = 1880 \text{ (People)}$$

Ex 4

$$y = Ae^{kt}$$

y - BALANCE
t - time
k - 3.1% (0.031)
A - FIND OUT

$1077 = Ae^{0.031t}$

$1077 = Ae^{0.031(0)}$ ← $y = 1077$, $t = 0$

$A = 1077$

$y = 1077e^{0.031t}$

$1486.73 = 1077e^{0.031t}$

$\frac{1486.73}{1077} = e^{0.031t}$

$1.38 = e^{0.031t}$ "ln" BOTH SIDES

4) A savings account balance is compounded continuously. If the interest rate is 3.1% per year and the current balance is $1077.00, in how many years will the balance reach $1486.73?

$\ln 1.38 = 0.031t$

$0.322 = 0.031t$

$t = 10.4$ years

Ex. 6

$$y = Ae^{kt}$$

y - POPULATION

t - TIME

K - RATE FACTOR

A - FIND OUT.

@ $t = 0$

$y = Ae^{k(0)}$

$y = A$ A EQUALS INITIAL POPULATION OF "y"

NOW DOUBLE "y" IN 20.4 min TO FIND K

$y = Ae^{kt}$

6) During the exponential phase, E. coli bacteria in a culture increase in number at a rate proportional to the current population. If the population doubles in 20.4 minutes, in how many minutes will the population triple?

$$2y = y e^{k(20.4)}$$

TRUE!

$$2 = e^{k(20.4)}$$

"ln" BOTH SIDES

$$\ln 2 = k(20.4)$$

$$k = \frac{\ln 2}{20.4}$$

$$k = 0.0339$$

$$y = A e^{0.0339t}$$

(POPULATION x 3)

$$3y = y e^{0.0339t}$$

$$3 = e^{0.0339t}$$

$$\ln 3 = 0.0339t$$

$$t = 32.4 \text{ min}$$