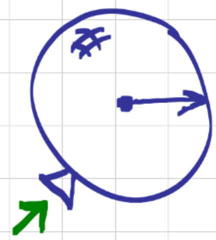
First two Related Rates Examples

EXAMPLE #1

A) DRAW A PICTURE.



BALLOON

$r = 50 \text{ cm}$

$$\frac{dV}{dt} = 1000 \text{ cm}^3/\text{s}$$

$$\max \frac{dr}{dt} = 0.029 \text{ cm/s}$$

B) LIST OR CREATE EQUATIONS THAT "LINK" VALUES OF INTEREST.

VOLUME (OF A SPHERE)

$$\boxed{V = \frac{4}{3}\pi r^3}$$

C) DERIVE THE EQUATION WITH RESPECT TO **TIME**

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3} 3\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

D) PLUG IN KNOWN VALUES AND SOLVE FOR UNKNOWNS:

$$1000 \text{ cm}^3/\text{s} = 4\pi (50)^2 \left(\frac{dr}{dt}\right)$$

(WATCH UNITS)

$$1000 \text{ cm}^3/\text{s} = 31416 \frac{dr}{dt}$$

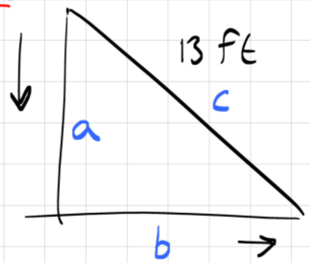
$$\frac{dr}{dt} = 0.032 \text{ cm/s}$$

ABOVE MAXIMUM RECOMMEND RATE!

NOT SAFE

Example 2

A)



IMPORTANT!
(−) DECREASING IN SIZE

- $\boxed{\frac{da}{dt} = -2\ ft/s}$
- FIND $\frac{db}{dt}$ (WHEN $a = 5$)

B) $c^2 = a^2 + b^2$ (LINKING EQUATION)

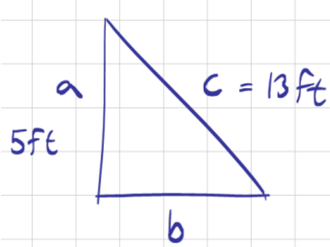
C) $2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$

D) $2(13)(0) = 2(5)(-2) + 2b\frac{db}{dt}$

LENGTH OF C NOT CHANGING
(NO RATE OF CHANGE)

$$0 = -20 + 2b\frac{db}{dt}$$

NEED TO FIND b SO WE CAN SOLVE FOR $\frac{db}{dt}$



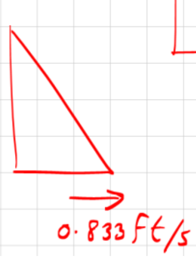
$c^2 = a^2 + b^2$
$(13)^2 = (5)^2 + b^2$
$b = 12\ ft$

$$0 = -20 + 2b\frac{db}{dt}$$

$$0 = -20 + 2(12)\frac{db}{dt}$$

$$0 = -20 + 24\frac{db}{dt}$$

$$\boxed{\frac{db}{dt} = 0.833\ ft/s}$$



LADDER SLIDING OUT @ 0.833 ft/s AT THE MOMENT DESCRIBED.