Optimization Example Solutions


WHAT DO WE WANT TO OPTMIIE?
(1) OBTELTIVE EQUATION
$\Rightarrow$ MAXIMIZE AREA

$$
A=(L) \times(W)
$$

(2) WHAT ARE THE CONSTRAINTS?

ONLY 40 ft OF FENCING AVAILABLE.

$$
2 W+L=40
$$

$\frac{\text { TOP VIEW }}{\frac{\text { BARN }}{\frac{\text { PEN }}{L}} \text { FENCE }}$
(3) sub constraint equation into okjectiut equation so OBJECTIVE EQUATION IS A FUNCTION OF ONLY ONE VARIABLE

$$
\begin{array}{rl}
2 W+L=40 & A=(L) \times(W) \\
L=40-2 W & A
\end{array} \begin{aligned}
& =(40-2 W)(W) \\
A & =40 W-2 W^{2}
\end{aligned}
$$

NOW WE HAVE A FUNCTION FOR:
AREA IN TERMS OF WIDTH
On the next page we will find what W needs to be to Maximize $\mathrm{A} . .$.

Let's find the value of w that gives us max A

$$
A=40 w-2 w^{2}
$$

$$
A^{\prime}=40-4 W
$$

(MAX or min possible

$$
\text { WHEN SLOPE }=0) \Longrightarrow 0=40-4 \mathrm{~W}
$$

$$
4 W=40
$$

$W=10$ POSSIBLE MAX

CONSTANT EQ.

$$
\begin{array}{r}
2 W+L=40 \\
2(10)+L=40 \\
L=20
\end{array}
$$

BUILD A loft $x 20 \mathrm{ft}$ PEN
for kitilies. (MAX AREA)


Example\#2
(1) OBJECTIUE EQUATION: MINIMUM SURFALE AREA "S.A."

(2) NEED A CONSTRAINT EQUATION TO GET SA IN TERMS OF A SINGLE VARIABLE.

VOLUME MUST BE 1000 mL or $1000 \mathrm{~cm}^{3}$ CONSTRAINED
By VOLUME

$$
1000=\pi r^{2} h \text { VOLUME OF A CyLINDER }
$$

(3) PUT constraint into objective.

$$
1000=\pi r^{2} h \quad S A=2 \pi r^{2}+2 \pi r h
$$

$h=\frac{1000}{\pi r^{2}}$

$$
\delta A=2 \pi r^{2}+2 \pi r \frac{1000}{\pi r^{2}}
$$

$$
\begin{aligned}
& S A=2 \pi r^{2}+ \\
& \text { MIN POINTS }
\end{aligned}
$$

(4) Find possible max min points

$$
\begin{aligned}
& S A^{\prime}=4 \pi r-2000 r^{-2} \\
& 0=4 \pi r-\frac{2000}{r^{2}} \quad(\text { MULTIPLYDOTH SIDS } \\
& 0=4 \pi r^{3}-2000 \\
& 2000 \\
& =4 \pi \Gamma^{3} \\
& r=5.42 \mathrm{~cm} \\
& \text { OTHER OpTIONS FOR } T \text { ? END POINTS? } \\
& f^{\prime}(x)=\text { D.N.E. ? } \\
& \therefore \min 5 A @ \\
& r=5.42 \mathrm{~cm} \\
& h=10.83 \mathrm{~cm}
\end{aligned}
$$

## Example\#3

## Box

OBTELTIUE EQUATION: MINIMIZE CAKOBOARD (SUAFALE AREA)

$S A=? ? ?$

SA $=w^{2}+4(w h)$ ObJECTVE
(2) CONSTRAINT VOLUME $=800 \mathrm{~mL}$


$$
\begin{aligned}
V & =w^{2} h \\
800 & =\omega^{2} h
\end{aligned}
$$

(3) put constrantint into
objective

$$
800=w^{2} h
$$

$$
h=\frac{800}{w^{2}}
$$

$$
S A=w^{2}+4(w h)
$$

$$
\delta A=w^{2}+4 w \frac{800}{w^{2}}
$$

$$
\begin{array}{ll}
\text { (4) FIND PoISIBLE } & \delta A=W^{2}+\frac{3200}{W} \\
\text { VALUES OF W } \\
\text { THAT GIUE MINS } & S A^{\prime}=2 W-3200 W^{-2}
\end{array}
$$



$$
\begin{aligned}
& 0=2 w-3200 w^{-2} \times W^{2} \\
& 0=2 w^{3}-3200 \\
& w^{3}=1600 \\
& w=11.7 \mathrm{~cm} \quad \text { (NO OTHEN Possioikitics } \\
& \text { of MIN?) } \\
& \text { NO. }
\end{aligned}
$$

$$
\begin{aligned}
& \left.800=w^{2} h \quad \begin{array}{l}
W=11.7 \mathrm{~cm} \\
\\
h=6 \mathrm{~cm}
\end{array}\right\} \begin{array}{l}
\text { FON MIN SUNFACL } \\
\text { AREA }
\end{array} 1.7
\end{aligned}
$$

