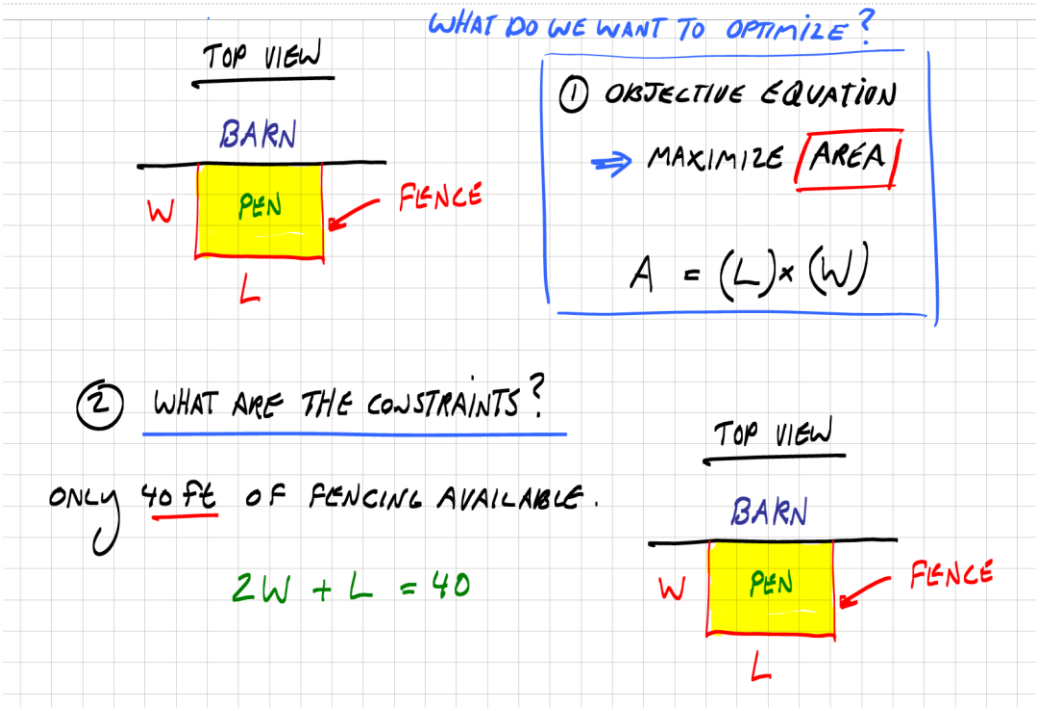
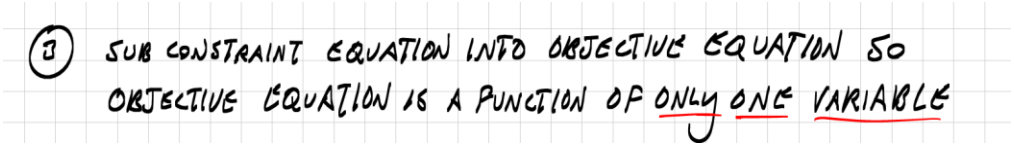
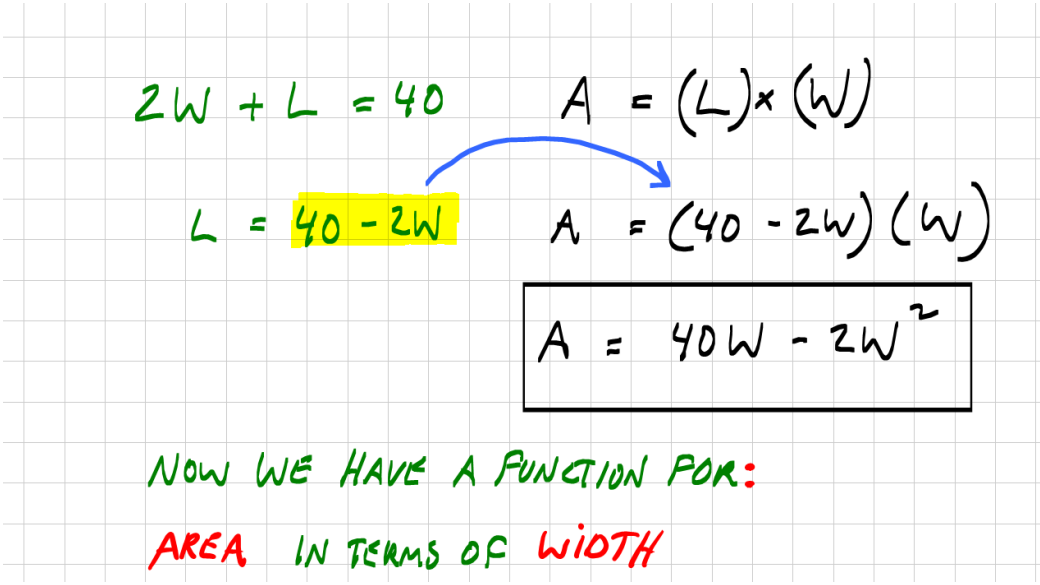
Optimization Example Solutions







On the next page we will find what W needs to be to Maximize A….

LET'S FIND THE VALUE OF W THAT GIVES US MAX A

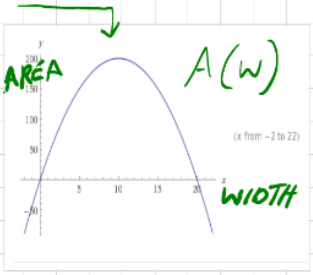
$$A = 40W - 2W^2$$

$$A' = 40 - 4W$$

(MAX OR MIN POSSIBLE WHEN SLOPE = 0) $\Rightarrow$

$$0 = 40 - 4W$$

$$4W = 40$$



$W = 10$ POSSIBLE MAX AREA WHEN WE HAVE A WIDTH OF 10 ft

CONSTRAINT EQ.

$2W + L = 40$

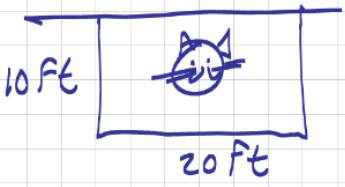
$2(10) + L = 40$

$L = 20$

OTHER POSSIBILITIES (ENDPOINTS ?.... $f'(x)$ D.N.E. ?)

NOPE.

BUILD A 10 ft x 20 ft PEN FOR KITTIES - (MAX AREA)



Example#2

① OBJECTIVE EQUATION: MINIMUM SURFACE AREA "S.A."

SURFACE AREA OF A CYLINDER

$$S.A. = 2(\pi r^2) + 2\pi r h$$

$2(\pi r^2)$: 2 CIRCLES TOP + BOTTOM

$2\pi r h$: SIDE OF CAN

② NEED A CONSTRAINT EQUATION TO GET SA IN TERMS OF A SINGLE VARIABLE.

VOLUME MUST BE 1000 mL or 1000 cm^3 CONSTRAINED BY VOLUME

$$V = \pi r^2 \cdot h$$

$$\boxed{1000 = \pi r^2 h}$$ VOLUME OF A CYLINDER

③ PUT CONSTRAINT INTO OBJECTIVE.

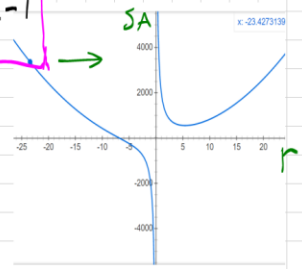
$1000 = \pi r^2 h$

$$h = \frac{1000}{\pi r^2}$$

$$SA = 2\pi r^2 + 2\pi r h$$

$$SA = 2\pi r^2 + 2\pi r \frac{1000}{\pi r^2}$$

$$SA = 2\pi r^2 + 2000 r^{-1}$$



④ FIND POSSIBLE MAX MIN POINTS

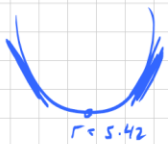
$$SA' = 4\pi r - 2000 r^{-2}$$

$$0 = 4\pi r - \frac{2000}{r^2}$$ (MULTIPLY BOTH SIDES BY r^2)

$$0 = 4\pi r^3 - 2000$$

$$2000 = 4\pi r^3$$

$$\boxed{r = 5.42 \text{ cm}}$$



OTHER OPTIONS FOR r? END POINTS? $f'(x)$ = D.N.E.? NOPE!

∴ min SA @

$r = 5.42$ cm

$h = 10.83$ cm

Example#3

BOX

OBJECTIVE EQUATION: MINIMIZE CARDBOARD (SURFACE AREA)

SA = ???

$SA = w^2 + 4(wh)$ OBJECTIVE

(2) CONSTRAINT VOLUME = 800mL

$V = w^2h$

$800 = w^2h$

(3) PUT CONSTRAINT INTO OBJECTIVE

$800 = w^2h$

$h = \frac{800}{w^2}$

$SA = w^2 + 4(wh)$

$SA = w^2 + 4w\frac{800}{w^2}$

$SA = w^2 + \frac{3200}{w}$

(4) FIND POSSIBLE VALUES OF w THAT GIVE MINS

$SA' = 2w - 3200w^{-2}$

$0 = 2w - 3200w^{-2}$ $\times w^2$

$0 = 2w^3 - 3200$

$w^3 = 1600$

$w = 11.7$ cm

(NO OTHER POSSIBILITIES OF MIN?)

NO.

$800 = w^2h$

$w = 11.7$cm
$h = 6$ cm

} FOR MIN SURFACE AREA