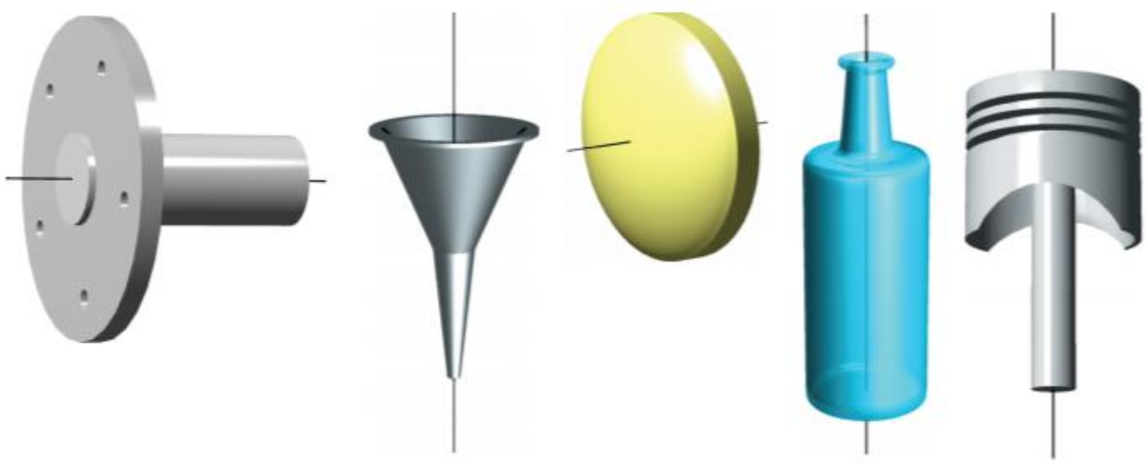
# Integral Applications
# Volume – The Disk Method

We mentioned earlier that area is *only one* of the many applications of integration.

Another important application is finding the **volume** of a three-dimensional solid.

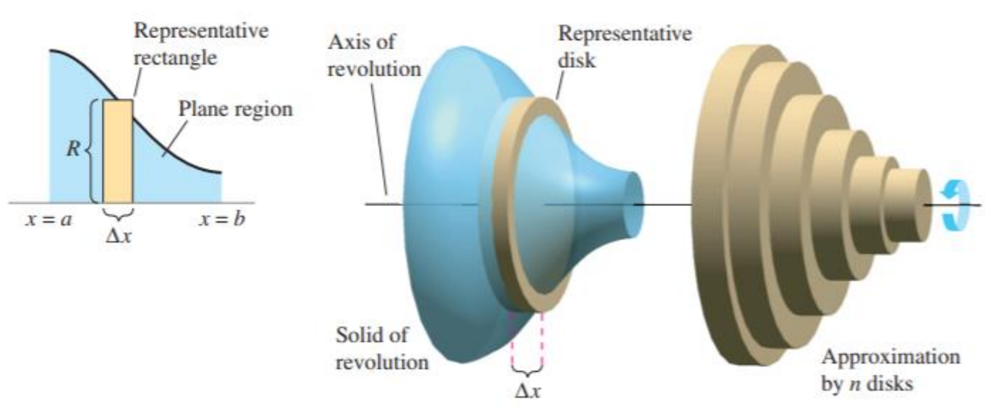
In this section you will study a particular type of three dimensional solid—one whose cross sections are always **circular.**

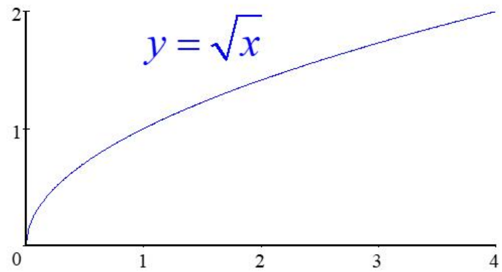
Objects of this sort are not uncommon in engineering and manufacturing. Some examples are axles, funnels, pills, bottles, and pistons, as shown below:

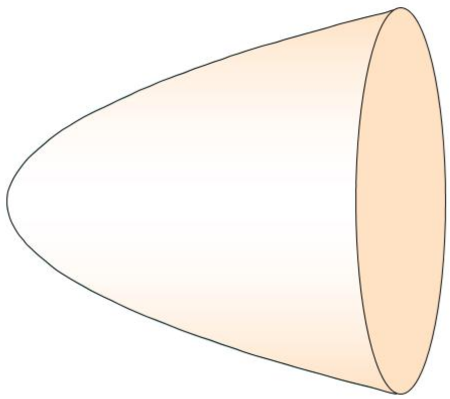


All **circular** cross-sections!

Before we integrated by adding up areas of rectangles.
Now we will ***add up Volumes of circular disks***





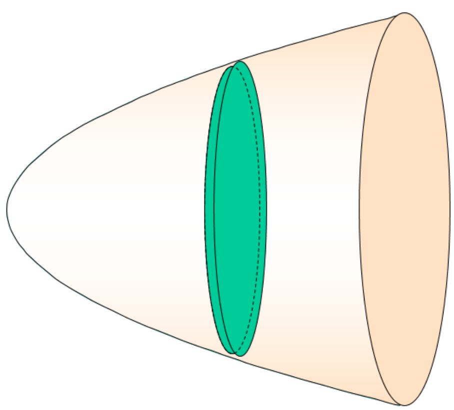
Your boss at SpaceX has asked you to construct a nose cone for a rocket.

The cone has been designed for least air resistance. It's profile matches the function shown.

**Let's examine how we might determine the Volume of the nose cone.**

Notice is has a circular cross-section

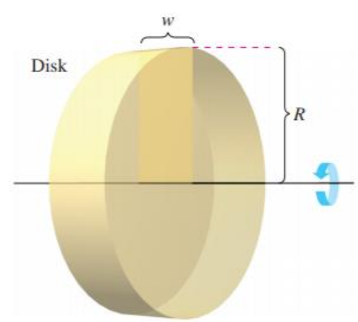
To do this let's try to add up ***cylindrical slices of volume*** using integration.



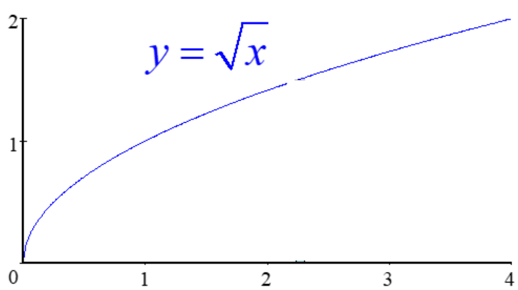
What is the **volume** of one of these slices?

Volume of **one** disk will be:

Volume of a disk: $\pi R^2 w$




In this case the ***width*** of each disk is ***dx***
And the ***radius*** R will be the ***y*** value of the function:




**The volume of each cylinder (disk) is:**

$\pi r^2$ **x (thickness of disk)**

$$\pi \left(\sqrt{x}\right)^2 \ dx$$

**If we add the disks, from 0 to 4 we get:**

$$\int_0^4 \pi\left(\sqrt{x}\right)^2 dx$$

This is a definite integral we can solve:

$$= 8\pi$$

Go tell the boss the volume will be 8π units.

The big take away from this exercise is that you can find the volume of circular coss-sectional objects $with\ a\ known\ profile$ by using:

Horizontal Axis of revolution:

$$V = \pi\int_a^b \left[f\left(x\right)\right]^2 dx$$

radius width